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COMPUTATION OF THE RESIDUAL RADIATION  
TRANSMISSION FACTOR FOR THE MARINE  
CORPS LVTP7

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20.

Abstract; (cont'd)

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The radiation reduction factors,  $R = P^{-1}$  for these two cases are 0.26 and 0.24 respectively; and the residual radiation transmission factors, defined from  $R$  to include the effects of a ground roughness parameter of 3 ft, are both 0.3.

The analytical method also should be useful to save computation time and cost in numerical work.

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by

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We develop an analytical technique for computing the gamma radiation reduction factor for a slab of arbitrary orientation in a field of uniformly distributed radioactive fallout ( $\text{Co}^{60}$ ). We apply this to a model of the army M113 APC, assuming the entire reduction is due to material in the vehicle walls, ceiling and floor. The resulting protection,  $P = 1.5$ , differs sharply with the measured value,  $P = 4.0$ , which we assume is due to the presence of other material than that in the vehicle surfaces. As a second model of the M113 we assume its shape to be that of a parallelepiped having a square vertical cross section of side  $L$ , uniform wall thickness  $t_{\text{eff}}$  and height equal to the actual vehicle height. We choose  $L$  and  $t_{\text{eff}}$  so that the surface area and total mass of the model vehicle agree with those of the actual vehicle. A 10% albedo correction to the protection then results in agreement with experiment:

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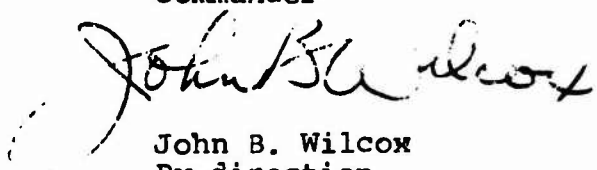
NOLTR 73-137

Computation of the Residual Radiation Transmission Factor for the Marine Corps LVTP7.

This report gives a new method for calculating the  $\gamma$ -radiation transmission through a plane slab of arbitrary orientation, above a field of uniformly distributed radioactive fallout. The method is applied to the Marine Corps LVTP7 to estimate the residual radiation transmission factor. The work was sponsored by the U. S. Marine Corps Development and Education Command.

The author wishes to thank M. A. Schmoke for a discussion, and for securing certain data pertaining to the army M113 APC, and C. M. Huddleston for discussions and for assistance in obtaining reference material, and for reading the report.

ROBERT WILLIAMSON II  
Captain, USN  
Commander



John B. Wilcox  
By direction

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## I. INTRODUCTION

The present work reports the results of an effort to compute the protection factor and the residual radiation transmission factor (RRTF: defined in Section 2) for the Marine Corps LVTP7 troop carrier in a field of radioactive fallout. The P7 is similar to another vehicle, the army M113 APC (armored personnel carrier), for which experimental data are available,<sup>1</sup> and we have used those results in our method. Briefly, we have developed an approximate analytical technique, which enables us to calculate the protection factor for an enclosure whose walls are composed of plane slabs having arbitrary orientations.

Applied to a fairly faithful model of the M113 geometry and using for wall thicknesses the actual values, the calculated value of the protection factor is  $P = 1.53$ , much lower than the experimental value of 4.0. The reason for this discrepancy is that the mass of the M113 in the model is less than half the mass of the actual vehicle in the (combat loaded) experimental conditions, so there is quite a bit more "extra" material actually present to enhance the protection over that afforded by the model; this material includes the engine, treads, wheels, axles, fuel, troops, interior panelling surfaces, and sundry paraphernalia, the mass distribution of which it is impractical to try to take into account. In the first place the labor involved would be enormous, and the results withal probably not trustworthy. Even more significantly, there is a "theoretical error" inherent in the notion of vehicle protection factor, a circumstance which makes too precise a result meaningless: the actual protection factor varies from place to place inside the vehicle owing to the irregular mass distribution (including shape), and to dependence of the radiation source intensity on height above the ground. Variations arising from these, and other effects as well, are at least 10 - 15%, experimentally,<sup>1</sup> so the complexity of the actual circumstances has the effect of easing the calculational problem.

To deal with the problem of the extra mass we have idealized the actual vehicle by a rectangular parallelepiped with square shape and having height equal to that of the actual vehicle. The wall thickness and the size of the square were fixed by requiring equality between the masses and surface areas of the actual vehicle and the model vehicle. The equation of the protection factor has an especially simple approximate form for this geometry, and the results we have obtained are in good agreement with experiment. Eqs. (6.18) and (6.17) give the values reported in Reference 1, respectively, for the "survey correlation factor," which, for the

experimental conditions of ground roughness, was .86 times the protection factor, and the calculated results from our model, which assumes a 10% albedo contribution to the reduction factor.

The results for the P7, modeled in this way, as a square rectangular parallelepiped, are given in Section 6 (eqs. (6.20) and (6.22)).

The outline of the report is this. In Section 2 we establish the character of the physical approach we have adopted, present source and buildup data, motivate our choice for ground roughness parameter, and give the general expressions of the protection factor, the survey correlation factor and the RRTF. In Section 3 we explicate the geometrical problem of the reduction factor (unscattered component) for a plane slab of arbitrary orientation, introducing a certain shape approximation which exploits essential features of the source. In Section 4 we develop the principal analytical formulas for the slab reduction factor and solid angle factor. In Section 5 we present the results of the M13 model calculation which neglects the "extra matter" actually present but not contained in the walls. In Section 6 we present details of the parallelepiped model and give results for both the M13 and the P7. In Section 7 we discuss the results, and in Section 8 we give a brief summary of what we have done and offer a concluding remark concerning application of our method to computer programming.

## 2. PRELIMINARY CONSIDERATIONS

A. Physical Approach

We assume the exposure at a point inside the vehicle to be a sum of contributions from all directions, with that from any given direction expressed as the product of a buildup factor with the exponentially attenuated unscattered component deriving from an assumed angular dependent source. For the source distribution we assume a form derived from curves given by Spencer,<sup>2</sup> for air, above an infinite plane of uniformly distributed fallout, simulating effects of ground roughness by using an effective height above the ground which is augmented by an amount  $\Delta h$  over the actual height.<sup>2</sup> The normalization of the angular distribution,  $\ell(h, x)$ , where  $x = \cos \theta$ ,  $\theta$  the angle of the line of sight from the detector position, of the incident  $\gamma$  ray with the downward vertical, is set by the usual condition,

$$L(h) = \int_{-1}^{+1} dx \ell(h, x) = 1, \text{ for } h = 3 \text{ feet} \quad (2.1)$$

To neglect of albedo, the reduction factor for detector height  $h$  ( $h_{\text{eff}} = h + \Delta h$  if there is ground roughness) due to a slab of thickness  $t_i$  and surface  $\Sigma_i$  is

$$r_i = r_{\Sigma_i}(h) = B_0(t_i) E_{\Sigma_i}(h, t_i) \quad (2.2)$$

where  $B_0(t_i)$  is the buildup factor for the given slab material, whose thickness is  $t_i$ , and  $E_{\Sigma_i}(t_i)$  is<sup>3</sup>

$$E_{\Sigma_i}(h, t_i) = \frac{1}{2\pi} \int d\Omega g_{\Sigma_i}(\Omega) \ell(h, \cos \theta) \exp(-\mu_{\Sigma_i} t_{\Sigma_i} \sec \theta_0), \quad (2.3)$$

where  $\mu_{\Sigma}$  is the inverse attenuation length for the substance of which  $\Sigma$  is composed,  $\theta_0$  is the angle formed by the incident  $\gamma$ -ray and the normal to  $\Sigma$ ,  $\theta$  is the so-called refraction parameter, which depends on the slab material,<sup>3</sup> and the "step" function  $g_{\Sigma}(\Omega)$  is unity if the line of sight of the direction indicated by  $\Omega$  intersects  $\Sigma$ , and is zero otherwise. In the same spirit the reduction factor inside the enclosure, at height  $h$ --above the ground is understood--is

$$R(h) = aR = a \sum_i r_{\Sigma_i}(h_{\text{eff}}), \quad (2.4)$$

where the summation extends over all the surfaces of the enclosure. In eq. (2.4) we have included an overall factor  $a$  to account for albedo, i.e. the contribution to the exposure from  $\gamma$  rays back-scattered from the interior faces of the enclosure. The protection factor within the enclosure at height  $h$  is defined by

$$P(h) = L(3 \text{ feet}) R^{-1} = R^{-1}, \quad (2.5)$$

by eq. (2.1); it is the ratio of the exposure in the standard unprotected position (3 feet above a smooth infinite plane of uniformly distributed fallout) to the exposure at height  $h$  within the enclosure under (assumed) given ground roughness conditions. The survey correlation factor,  $S$ , reported in Reference 1, is a similar ratio, with the measured exposure at  $h = 3$  feet replacing that for the standard unprotected position; so

$$S(h) = L(3 \text{ feet} + \Delta h) R^{-1} = L(3 \text{ feet} + \Delta h) P. \quad (2.6a)$$

Since  $L(h)$ , defined in eq. (2.1), is a decreasing function of  $h$ ,  $S(h)$  is smaller than  $P(h)$ . We define the RRTF to be  $S^{-1}$ :

$$\text{RRTF} = S^{-1}. \quad (2.6b)$$

### B. The Source

If the source were that due solely to unscattered  $\gamma$ -rays of energy  $E$ , we would have

$$\begin{aligned} l(h,x) &= c x^{-1} \exp(-\mu_a h x^{-1}), & x > 0, \\ l(h,x) &= 0 & x < 0, \end{aligned} \quad (2.7)$$

where  $\mu_a^{-1}$  is the  $\gamma$ -ray attenuation length in air for energy  $E$ , and  $c$  is a normalization coefficient. In Figures 1 and 2 we have reproduced two of the curves from Reference 2, plots of  $l(h,x)$  for  $h = 6.6$  feet and  $h = 66$  feet, together with plots of eq. (2.7) for values of  $\mu_a$  such that the location of the peaks of the former coincide with those of the latter, which occur at

$$x = \mu_a h \equiv x_0, \quad (2.8)$$

and with normalization of (2.7) chosen so that the heights of the peaks coincide. For  $h = 6.6$  feet the agreement for  $x > 0$  is extremely good and for  $h = 66$  feet the disagreement is of order 10%, or so. Our interest is in height values appropriate to the 6.6 feet case.

To very good accuracy, to quite a bit better than 5% for heights of interest to us, we are justified in using the form (2.7) for  $x > 0$ . For the skyshine part,  $x < 0$ , we assume a parabola centered at  $x = -1$ ,

$$l(h,x) = c_1 + c_2 (x + 1)^2, \quad (2.9)$$

requiring the constants  $c_1$  and  $c_2$  to be chosen so that

$$l(h,-1)/l(h,0) = .195 \quad (2.10)$$

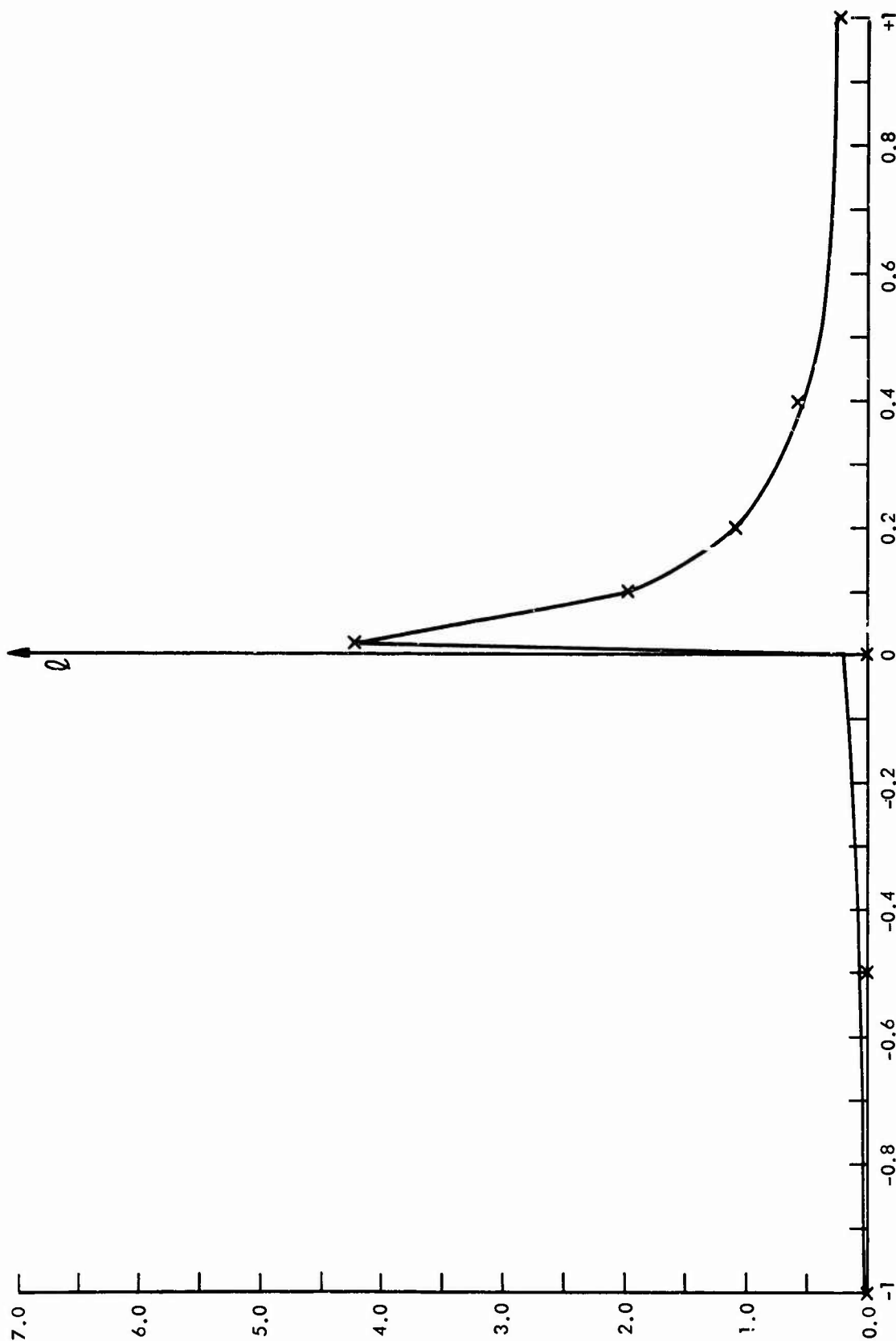


FIG. 1 Exposure angular distribution ( $x = \cos \theta$ ) above an infinite plane of radioactive fallout. Solid curve (taken from reference 2): in water, scaled to air, height 6.6 ft.; 1.12 hr. fission spectrum. The crosses show the function given in eq. (2.7), with  $h = 6.6$  ft., and  $\mu_g$  and  $c$  chosen so that the peak of the corresponding curve coincides with the peak of the curve drawn in the figure.

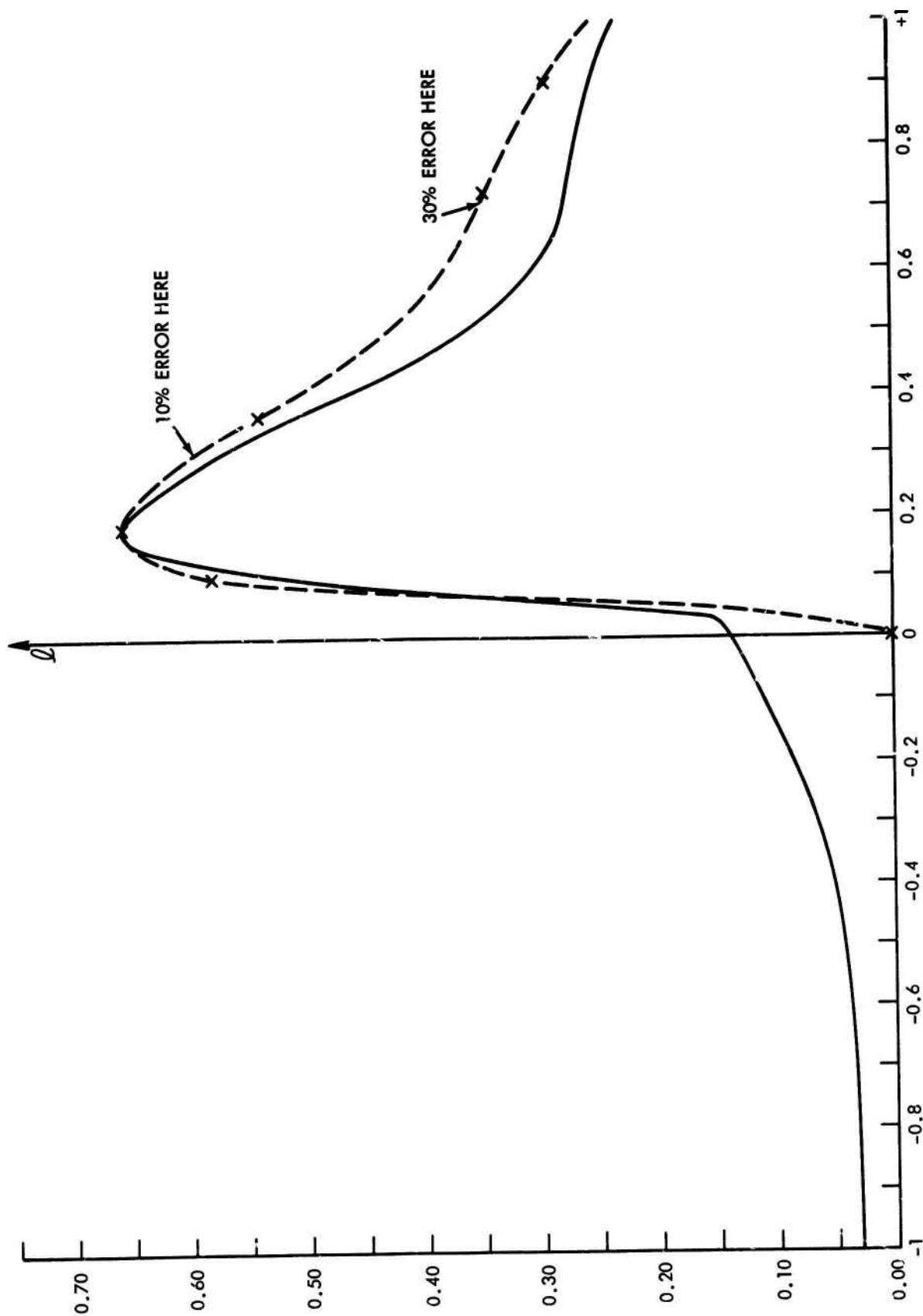


FIG. 2 Same as FIG. 1, but with  $h = 66$  ft. Here the crosses have been joined by a dashed curve.

which is good to <5% between  $h = 3.3$  feet and  $h = 33$  feet. We fix the relative normalization between the  $x < 0$  and  $x > 0$  parts by requiring

$$S = .110L, \quad (2.11)$$

where

$$S = \int_{-1}^0 dx \ell(h, x), \quad (2.12)$$

is the skyshine contribution to  $L$ . Between 3 feet and 33 feet  $S$  rises from .093 $L$  to .125 $L$ , from curves in Reference 2; interpolation gives  $S = .105L$  for  $h = 6.6$  feet, while direct measurements from Figure 1 have given us eq. (2.11).

The normalization coefficient  $c$ , in eq. (2.7), still remains to be fixed; it is a buildup factor, and it depends on  $h$ . Also, we need to choose a value for  $\mu_a$ . First the latter: the contribution to  $L$  from  $x > 0$ , assuming the form in eq. (2.7), is  $cE_1(\kappa)$  where  $E_1(\kappa)$  is the exponential integral,

$$E_1(\kappa) = \int_{\kappa}^{\infty} dy y^{-1} \exp(-y); \quad (2.13)$$

we equate this at  $h = 6.6$  feet to

$$L(6.6 \text{ feet}) - S(6.6 \text{ feet}) = .890L(6.6 \text{ feet}),$$

taking  $L(6.6 \text{ feet}) = .87$  from Spencer<sup>2</sup>, to get

$$c(6.6 \text{ feet})E_1(\kappa(6.6 \text{ feet})) = .774. \quad (2.14)$$

But, with  $x = \kappa$  locating the peak, we have also

$$c \cdot (e\kappa)^{-1} = \ell_p \quad (2.15)$$

where  $\ell_p$  is the peak value of  $\ell$ , which is 4.15 for  $h = 6.6$  feet; combining these gives

$$\kappa E_1(\kappa) \big|_{6.6 \text{ feet}} = .774 \cdot (4.15e)^{-1} = .0686, \quad (2.16)$$

whose solution is  $\kappa = .0207$  (to about 3%), so that\*

$$\mu_a^{-1} = 6.6 \text{ ft.} \div .0207 = 319 \text{ feet, or } 97\text{m,}$$

$$\mu_a = .003134 \text{ ft.}^{-1} \quad (2.17)$$

---

\* The value 97m seems a little low for the spectrum from which Spencer's curves are calculated; this was a 1.12 hr. fission spectrum, for which, using spectrum data given in Reference 2, we have estimated  $\mu_a^{-1} \approx 123\text{m}$ , to perhaps 15%.

If we use eq. (2.15), taking  $x = \mu_a h$  and reading  $l$  from Spencer's curves, we obtain a set of values of  $c$  which can be fit with a straight line

$$c = .2169 + .835 x, \quad 4 \text{ ft} \leq h \leq 42 \text{ ft.}, \quad (2.18)$$

while for  $h = 3 \text{ ft.}$  we get

$$c(3 \text{ ft.}) = .2172 \quad (2.19)$$

From eqs. (2.10), (2.11) and (2.18) we obtain  $c$ , and  $c_2$ , and with this the complete form for  $l(h, x)$ ,

$$l(h, x) = c \cdot \begin{cases} x^{-1} \exp(-\mu_a h x^{-1}), & x > 0, \\ .264 E_1(\mu_a h) [.195 + .805 (x+1)^2], & x < 0. \end{cases} \quad (2.20)$$

This can be used for  $4 \text{ ft.} \leq h \leq 42 \text{ ft.}$  even though (2.10) and (2.11) weaken above  $30 \text{ ft.}$ , for the whole skyshine contribution is not much more than 15 to 20% of the total below  $h = 42 \text{ ft.}$  When we integrate (2.20) using (2.19), we find  $L(3 \text{ ft.}) = 1.000$ .

The source function, eq (2.20), is not quite what we want, however. We need to shift the value of  $\mu_a$  if we wish to compare with experiments of Reference 1, where  $\text{Co}^{60}$  was used as the  $\gamma$ -ray source. The  $\text{Co}^{60}$  spectrum contains two principle peaks, at  $1.17 \text{ MeV}$  and  $1.33 \text{ MeV}$ . If we idealize this as a single peak at  $1.25 \text{ MeV}$ , the attenuation length in air is

$$\mu_a^{-1} = \frac{A}{Z} \cdot \rho^{-1} \frac{m_p}{f(E) \sigma_{th}} \quad (2.21)$$

where  $A/Z = 2$  is the atomic weight to number ratio for air,

$\rho = 1.312 \times 10^{-3} \text{ gm/cm}^3$  is the density of air (STP),  $m_p$  the mass of the proton,  $\sigma_{th}$  the Thompson cross section,

$$\sigma_{th} = \frac{8\pi}{3} r_o^2, \quad r_o = e^2 / m_e c^2 = \text{classical electron radius},$$

and  $f(E)$  is the Compton cross section for  $\gamma$  energy  $E$ , in units of  $\sigma_{th}$ . We get

$$\begin{aligned} \mu_a^{-1}(1.25 \text{ MeV}) &= 131.7 \text{ m} = 431 \text{ ft} \\ \mu_a &= .002314 \text{ ft}^{-1}, \end{aligned} \quad (2.22)$$

which are somewhat different from the values appearing in (2.17).

To convert to the  $\text{Co}^{60}$  spectrum, we simply take over eq. (2.20), regarding the right side as a function of  $\kappa = \mu_a h$ , valid between  $\kappa_1 = .003134 \text{ ft}^{-1} \times 4 \text{ ft} = .01254$ , and  $\kappa_2 = .003134 \text{ ft}^{-1} \times 42 \text{ ft} = .1316$ , where the  $\kappa$  dependence of  $c$  is indicated by eq. (2.18). There is an overall normalization change affecting the coefficients in eq. (2.18), however, arising from the fact that the normalization condition, eq. (2.1), holds for any  $\mu_a$ . We have, at  $h = 3 \text{ ft.}$ ,

$$c(3 \text{ ft}, .002314 \text{ ft}^{-1}) \equiv N \cdot c(3 \text{ ft}, .003134 \text{ ft}^{-1}) \quad (2.23a)$$

while from eqs. (2.1) and (2.20) with  $\mu_a = .002314 \text{ ft}^{-1}$ ,

$$c(3 \text{ ft}, .002314 \text{ ft}^{-1}) \times 1.1222 E_1(.006942) = 1 \quad (2.23b)$$

whence

$$N = .938 \quad (2.24)$$

We assume that the same factor scales the source to the new value of  $\mu_a$  for other values of  $h$  and we get

$$\begin{aligned} c &= .2034 + .783\mu_a h, & 5.5 \text{ ft.} < h < 55 \text{ ft.}, \\ &= .2038, & h = 3 \text{ ft.} \end{aligned} \quad (2.25)$$

The limits of validity to (2.25) are determined as  $\kappa_1/\mu_a$  and  $\kappa_2/\mu_a$  (compare eq. (2.18)).

Summarizing, the source is given in eqs. (2.20) and (2.25), for the case of a  $\text{Co}^{60}$  source.

### C. Buildup Factor for Transmission through a Wall.

Taking the data of Reference 3, we can fit the buildup factor, as a function of (normal) wall thickness, to a straight line. For  $X$  out to 2 mean free paths (which is all we need) we have, for concrete, or aluminum

$$B_o = 1 + .76X, \quad (2.26)$$

when  $X$  is the mass thickness in units of  $\mu^{-1}$ . For aluminum, using eq. (2.21) with  $\rho = 2.72 \text{ gm/cm}^3$ , we have  $\mu^{-1} = \mu_{Al}^{-1} = 2.60$  in, so that

$$B_o = 1 + .292 t \quad (2.27)$$

where  $t$  is the wall thickness in inches. This is good to a percent or so.

#### D. Ground Roughness

The data of Reference 1 permit the determination of the ground roughness parameter,  $\Delta h$ , which existed under the conditions of the experiment. Figure 3 shows a plot of the reported unprotected exposure as a function of height  $h$ , normalized to unity at 3 feet, and Figure 4 shows four theoretical curves of  $L(h+\Delta h)$ , for  $\Delta h = 0, 3, 6$ , and 9 feet, with zeros of ordinate chosen so that all curves pass through the same point at  $h = 3$  feet. When the two figures are superposed, the experimental curve lies on top of the theoretical curve having  $\Delta h = 3$  feet. For comparison purposes therefore we adopt this value for the ground roughness parameter throughout. We notice that if  $h < 5$  feet then  $\kappa < .002314 \times (5+3) = .0185 < 1$ , a fact we exploit below.

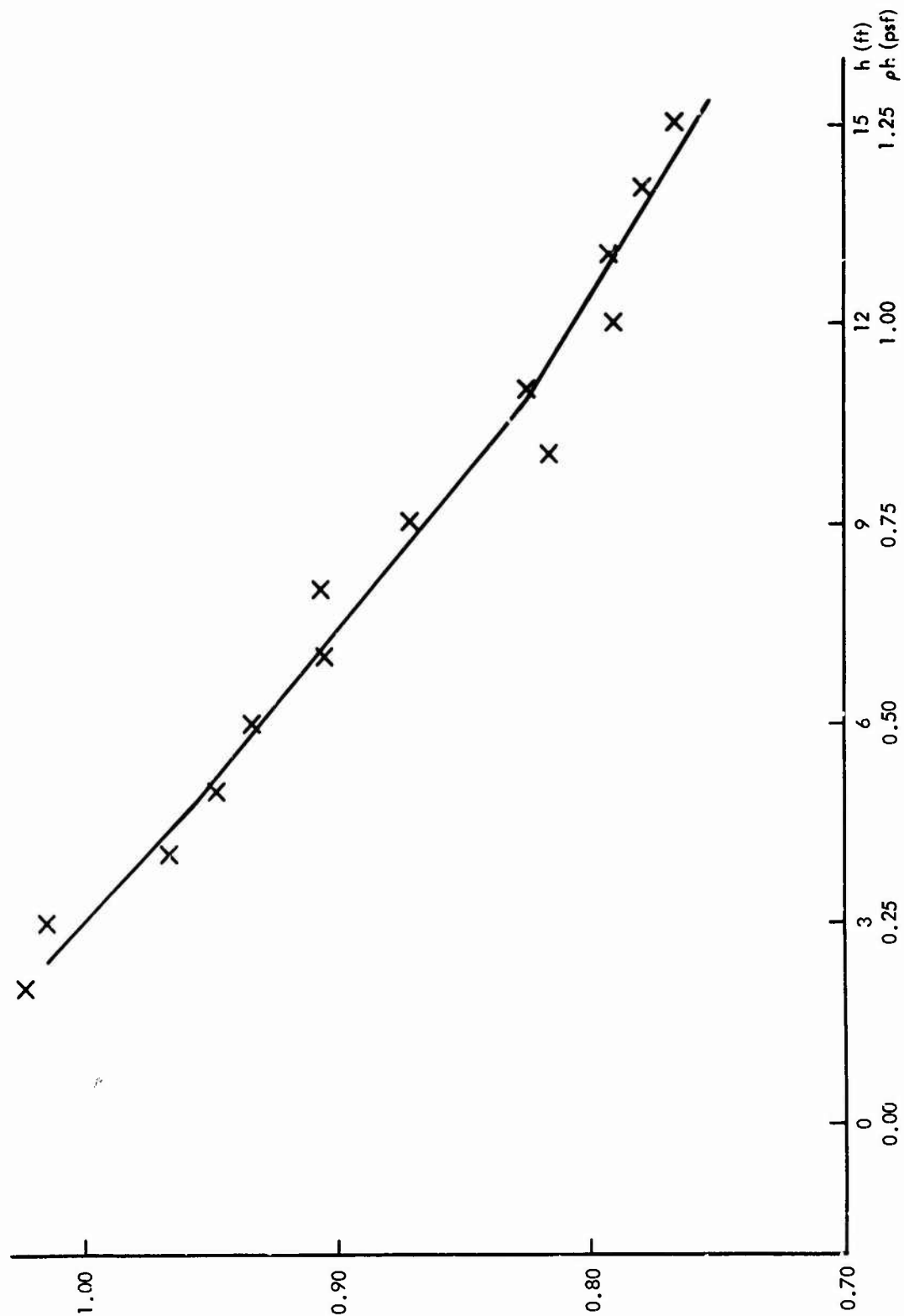


FIG. 3 Measured unprotected total exposure dose reported in reference 1, as a function of actual height above ground, and normalized (arbitrarily) to one, at 3 ft. (Co60 source).

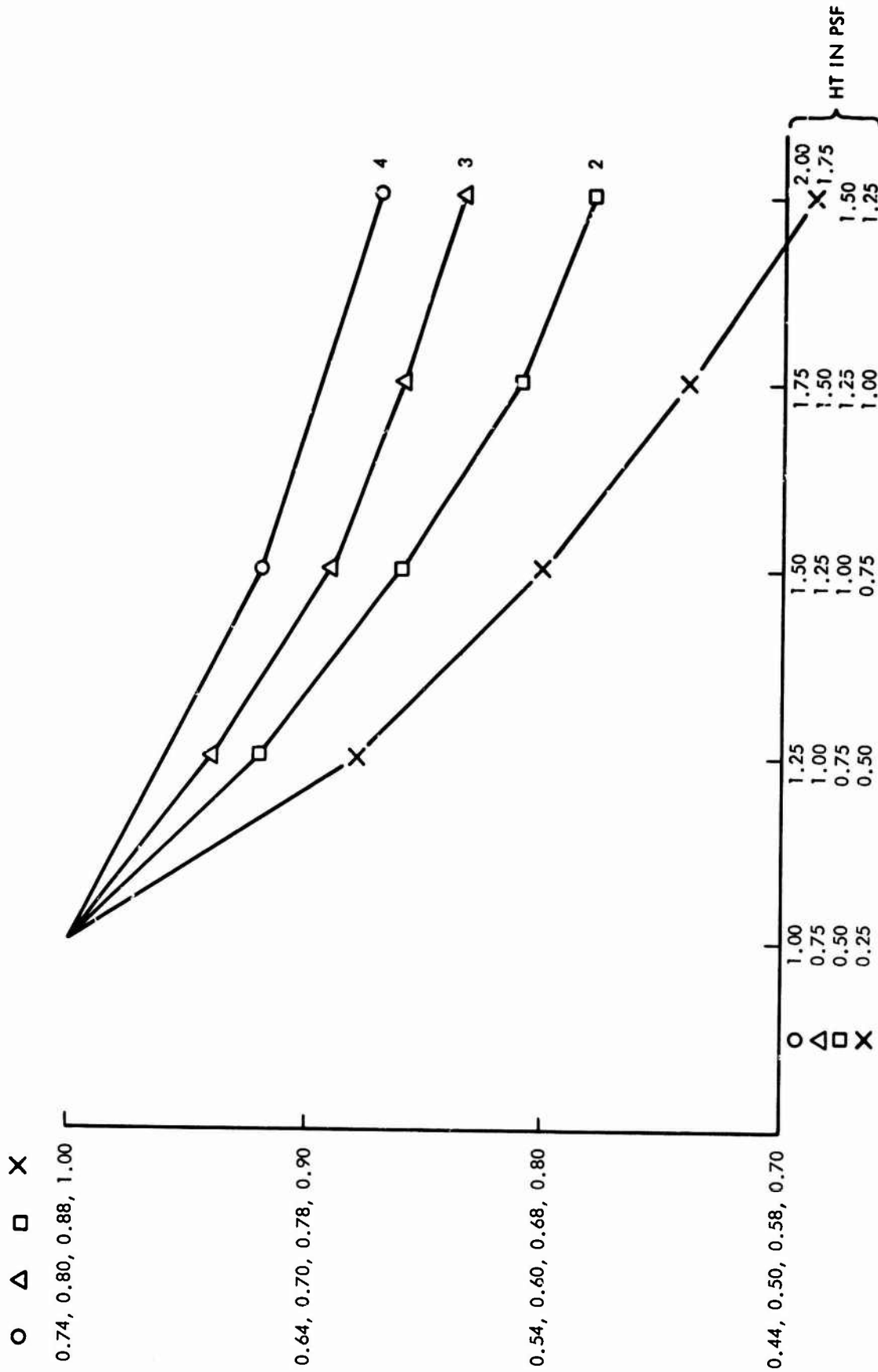


FIG. 4 Various segments of the curve of theoretic exposure vs matter thickness (height) above ground, plotted so that all curves intersect in a single point, that for which  $L$  (3ft) = 1.00. The data of FIG. 3 lie along curve 2, indicating a ground roughness,  $\Delta h = 3$  ft. (conversion formula:  $0.25 \text{ psf} = 3 \text{ ft}$ ).

## 3. PLANE SLAB HAVING ARBITRARY ORIENTATION

A. Geometrical Preliminaries

We choose cartesian coordinates  $(x_o, y_o)$  in the plane of the slab,  $\Sigma$ , with the  $\hat{x}_o$  - axis\* horizontal and the  $\hat{y}_o$  - axis forming an angle  $\alpha$  with the vertical, where  $\alpha = +\frac{\pi}{2}$  for the floor (Figure 5). The origin of coordinates,  $O$ , is the intersection of the perpendicular to the  $x_o y_o$  - plane from the detector position  $D = D(0,0,z_1)$  where  $z_1 > 0$  is the distance from slab to detector; the detector is positioned at height  $h$  above the horizontal plane of the ground. In addition we employ spherical coordinates, with origin at  $D$  and polar axis pointing down. The co-latitude angle of the point  $P(x_o, y_o, 0)$  at which the incident  $\gamma$ -ray passes through  $\Sigma$  is  $\theta$ , by the definition given above eq. (2.1). The unit vector which points from  $D$  to  $P$  is  $\hat{n}$  and the azimuth of  $\hat{n}$  in the plane parallel to the ground (through  $D$ ) measured from  $+\hat{x}_o$  counter-clockwise from above, is  $\varphi$ , the second polar variable of the point\*\*  $P$ . In Figure 5 we have included also an auxiliary (primed) set of cartesian axes, having the same origin  $O$  and  $\hat{y}'_o$  - axis pointing vertically upwards. From

$$\hat{n} = -\hat{y}'_o \cos \theta + (\hat{x}'_o \cos \varphi - \hat{z}'_o \sin \varphi) \sin \theta \quad (3.1)$$

$$\hat{x}'_o = \hat{z}_o \quad (3.2a)$$

$$\hat{y}'_o = -\hat{z}_o \sin \alpha + \hat{y}_o \cos \alpha, \quad (3.2b)$$

$$\hat{z}'_o = \hat{z}_o \cos \alpha + \hat{y}_o \sin \alpha \quad (3.2c)$$

we have

$$\begin{aligned} \hat{n} = & \hat{x}_o \cos \varphi \sin \theta - \hat{y}_o (\cos \varphi \cos \alpha + \sin \theta \sin \alpha \sin \varphi) \\ & + \hat{z}_o (\cos \varphi \sin \alpha - \sin \theta \cos \alpha \sin \varphi). \end{aligned} \quad (3.3)$$

---

\*Generally, we use the carat to denote a direction (unit) vector

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\*\* The spherical coordinate system is left-handed.

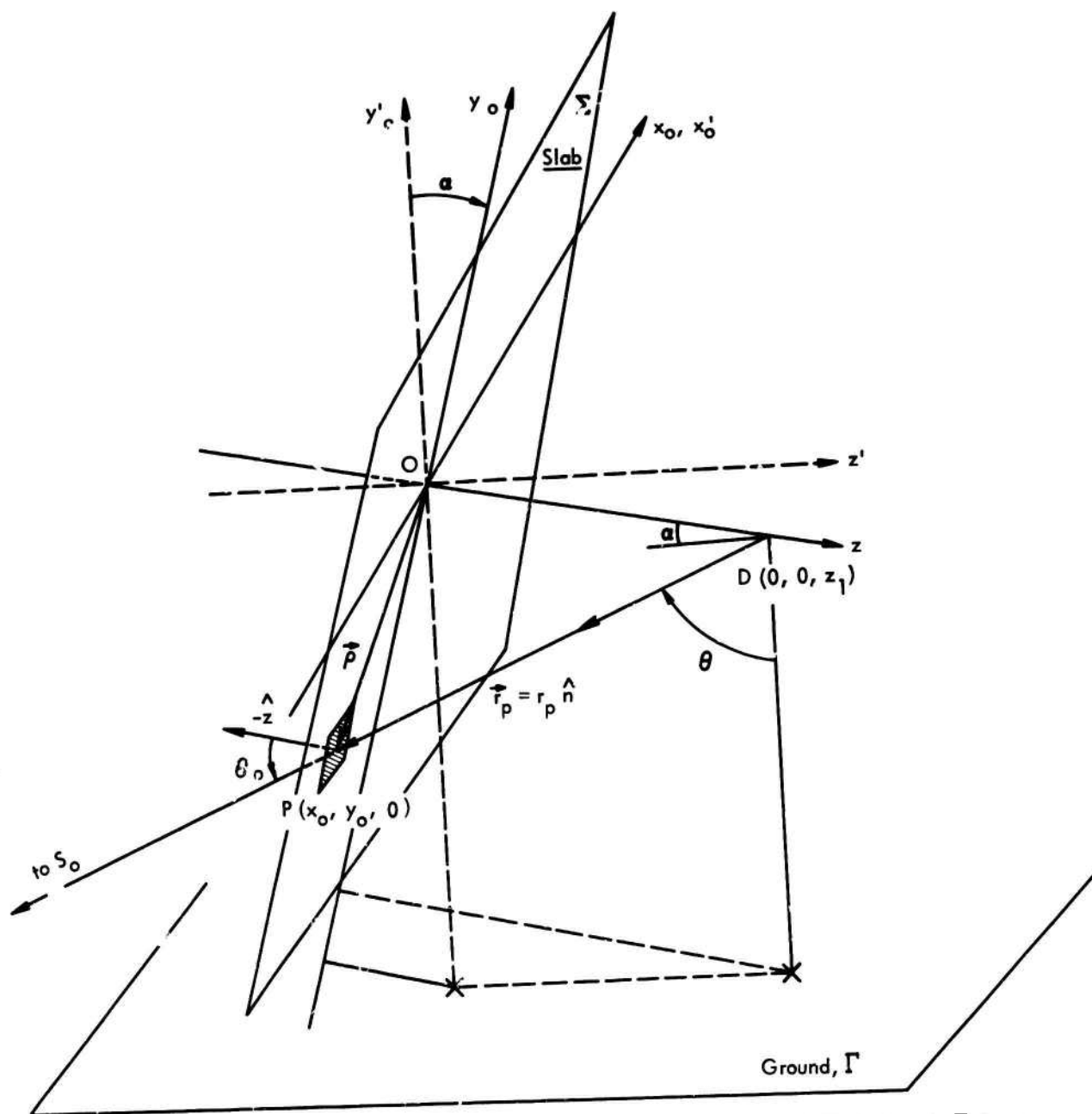


FIG. 5 Geometry of slab ( $\Sigma$ ) orientation, detector (D) position,  $\gamma$ -ray path (PD) through  $\Sigma$  from the ground,  $\Gamma$ . For the configuration shown,  $\alpha > 0$ .

We denote the vector  $\vec{OP}$  by  $\vec{\rho}_P$  and  $\vec{PD}$  by  $\vec{r}_P$  so that

$$\vec{r}_P = r_P \hat{n} = \vec{\rho}_P - z_1 \hat{z}_O \quad (3.4)$$

and

$$\vec{\rho}_P = x_O \hat{x}_O + y_O \hat{y}_O \quad (3.5)$$

$$r_P = (x_O^2 + y_O^2 + z_1^2)^{1/2}, \quad (3.6)$$

whence, using eq. (3.4), we find

$$x = x(x_O, y_O) = \cos \theta = -r_P^{-1} y_O \cos \alpha - r_P^{-1} z_1 \sin \alpha \quad (3.7)$$

$$\tan \varphi = -x_O^{-1} y_O \sin \alpha + x_O^{-1} z_1 \cos \alpha, \quad (3.8)$$

and

$$\frac{\partial(x, \varphi)}{\partial(x_O, y_O)} = -z_1 r_P^{-3}, \quad (3.9)$$

so that integrals of the type appearing in eq. (2.3) can be performed on the plane of the slab,

$$\frac{1}{2\pi} \int d\Omega_{\Sigma}(\Omega) \dots = \frac{1}{2\pi} \int_{x_1}^{x_2} \int_{y_1}^{y_2} dx_O dy_O \cdot \frac{z_1}{r_P^3} \dots, \quad (3.10)$$

where the "left" and "right" edges of  $\Sigma$  are located at  $x_O = x_1$  and  $x_O = x_2$ , respectively, and the "lower" and "upper" edges by  $y_O = y_1$  and  $y_O = y_2$ , respectively. If we let

$$\xi = x_O z_1^{-1}, \quad \eta = y_O z_1^{-1}, \quad (3.11)$$

then for (3.10) we may write

$$\frac{1}{2\pi} \int d\Omega_{\Sigma}(\Omega) \dots = \frac{1}{2\pi} \int_{\xi_1}^{\xi_2} d\xi \int_{\eta_1}^{\eta_2} d\eta (\xi^2 + \eta^2 + 1)^{-3/2} \dots, \quad (3.12)$$

where  $\xi_i = x_i z_1^{-1}$  and  $\eta_i = y_i z_1^{-1}$ ,  $i = 1, 2$ .

We also need to express  $\theta_O$  in terms of the cartesian variables; from

$$\cos \theta_O = \hat{n} \cdot (-\hat{z}_O), \quad (3.13)$$

with eq. (3.3),

$$\cos \theta_O = -\cos \theta \sin \alpha + \sin \theta \cos \alpha \sin \varphi, \quad (3.14)$$

whence with the aid of eqs. (3.7) and (3.8)

$$\cos \theta_0 = (\xi^2 + \eta^2 + 1)^{-\frac{1}{2}}, \quad (3.15)$$

which also can be seen by inspection (Figure 3).

### B. Basic Geometrical Approximation Scheme

There are two classes of slab orientation we need to consider:

- (i)  $|\alpha|$  well away from  $\frac{\pi}{2}$ , as in the case that  $\Sigma$  is a wall; and
- (ii)  $\alpha = \pm \frac{\pi}{2}$ , where  $\Sigma$  is a ceiling or a floor. For the latter we adopt the approximation scheme described in Reference 2, which replaces  $\Sigma$  by another slab,  $\Sigma_0$ , formed of suitable combinations of circles and annular sectors -- so we discuss only case (i) here.

The integrand of eq. (2.3) is peaked at  $x = \kappa$  owing to the form of  $l(h, x)$ . The intersection of the plane of  $\Sigma$  and the cone of constant  $x$  with vertex at the detector is a conic section, and  $l(h, x)$  is constant along this curve. We exploit this fact by transforming integrations over  $\Sigma$  to elliptical coordinates,  $u, v$ , in the plane of  $\Sigma$ , so chosen that the curve  $x = \kappa$ , which (normally) is a hyperbola, coincides with one of the curves of constant  $v$ :  $v = v_1$ . From eq. (3.7)

$$x = -(\xi^2 + \eta^2 + 1)^{-\frac{1}{2}} (\eta \cos \alpha + \sin \alpha), \quad (3.16)$$

so the hyperbola, which results when  $\cos^2 \alpha > \kappa^2$  is

$$a^{-2}(\eta + b)^2 - c^{-2} \xi^2 = 1 \quad (3.17)$$

where, for  $x = \kappa$

$$b = \sin \alpha \cos \alpha (\cos^2 \alpha - \kappa^2)^{-1} \quad (3.18)$$

$$\doteq \tan \alpha + O(\kappa^2), \quad \kappa \ll 1, \quad (3.18')$$

$$a^2 = \kappa^2 (1 - \kappa^2) (\cos^2 \alpha - \kappa^2)^{-2} \quad (3.19a)$$

$$c^2 = (1 - \kappa^2) (\cos^2 \alpha - \kappa^2)^{-1}. \quad (3.19b)$$

The angle  $|\Psi|$  formed by the  $\hat{x}_0$  - axis and the asymptotes to (3.17) is given by

$$\tan |\Psi| = a/c = \kappa (\cos^2 \alpha - \kappa^2)^{-\frac{1}{2}} \quad (3.20)$$

$$|\Psi| \doteq \kappa + O(\kappa^3), \quad \kappa \ll 1. \quad (3.20')$$

Half the distance between the foci,  $P_1$  and  $P_2$ , is

$$\begin{aligned}
D &= z_1 (a^2 + c^2)^{\frac{1}{2}} \\
&= z_1 (1 - \kappa^2)^{\frac{1}{2}} \cos \alpha (\cos^2 \alpha - \kappa^2) \\
&\doteq z_1 \sec \alpha + O(\kappa^2), \quad \kappa \ll 1.
\end{aligned}
\tag{3.21}$$

$$\tag{3.21'}$$

We introduce the elliptical coordinates as follows. We let  $r_1$  and  $r_2$  be distances of  $P(x, y_0)$  from  $P_1(0, -bz_1 - D)$  and  $P_2(0, -bz_1 + D)$  and define

$$u = (2D)^{-1}(r_1 + r_2), \quad v = (2D)^{-1}(r_1 - r_2). \tag{3.22}$$

We need the following relations, which follow from the above:

$$\eta = Dz_1^{-1} uv - b \tag{3.23}$$

$$\begin{aligned}
\sec \theta_0 &= (\xi^2 + \eta^2 + 1)^{\frac{1}{2}} \\
&= Dz_1^{-1} [u^2 - 2v_0 uv + v^2 - v_1^2(1 - v_0^2)]^{\frac{1}{2}},
\end{aligned}
\tag{3.24}$$

where

$$v_0 = \sin \alpha \text{ and } v_1 = -\kappa \sec \alpha, \tag{3.25}$$

the latter being the value of  $v$  for which  $x = \kappa$  holds; also

$$x = -\cos \alpha (uv - v_0 v_1^2) [u^2 - 2v_0 uv + v^2 - v_1^2(1 - v_1^2)]^{-\frac{1}{2}}, \tag{3.26}$$

$$\frac{\partial(\xi, \eta)}{\partial(u, v)} = \left(\frac{D}{z_1}\right)^2 (u^2 - v^2) [(u^2 - 1)(1 - v^2)]^{-\frac{1}{2}}. \tag{3.27}$$

For the approximation to small  $\kappa$ , we drop the  $v_1^2$  terms in eqs. (3.24) and (3.26),

$$\sec \theta_0 \doteq \sec \theta_0 (u^2 - 2v_0 uv + v^2)^{\frac{1}{2}} \tag{3.24^1}$$

$$x \doteq -uv \cos \alpha (u^2 - 2v_0 uv + v^2)^{-\frac{1}{2}} \tag{3.26^1}$$

For eq. (3.10) we have

$$\begin{aligned}
\frac{1}{2\pi} \int d\Omega g_{\Sigma}(\Omega) \dots &\doteq \frac{\cos \alpha}{2\pi} \iint_{\Sigma} du dv (u^2 - v^2) [(u^2 - 1)(1 - v^2)]^{-\frac{1}{2}} \\
&\times (u^2 - 2v_0 uv + v^2)^{-3/2} \dots,
\end{aligned}
\tag{3.28}$$

where the  $\cos \alpha$  in front comes from the scale factor  $(D/z_1)^2 \cdot (D/z_1)^{-3}$  (cf. eqs. (3.24) and (3.27)).

The integrand of  $E_{\Sigma}$ , in eq. (2.3), is proportional to

$$l(h,x) \exp(-a_{\Sigma} \sec \theta_0)$$

where

$$a_{\Sigma} = \mu_{\Sigma} t_{\Sigma}. \quad (3.29)$$

We approximate this by taking  $\beta \approx 1$ :

$$\sec \beta \theta_0 = \sec \theta_0 - (1-\beta) \theta_0 \sec \theta_0 \tan \theta_0 + \dots,$$

so

$$e^{-a_{\Sigma} \sec \beta \theta_0} = e^{-a_{\Sigma} \sec \theta_0 (1 + (1-\beta) \theta_0 \sec \theta_0 \tan \theta_0 + \dots)} \quad (3.30)$$

$$\approx e^{-a_{\Sigma} \sec \theta_0} \quad (3.30')$$

The approximation to (3.30') is reasonable if  $|\alpha|$  is not too large, so that  $\theta_0$  is relatively small in regions where  $l(h,x)$  peaks. For  $x > 0$  we have

$$l(h,x) e^{-a_{\Sigma} \sec \theta_0} = c x^{-1} \exp[-x^{-1}(\kappa - a_{\Sigma} uv)], \quad x > 0, \quad (3.31)$$

by eqs. (2.20), (3.24'), (3.26'), and for  $x < 0$ ,

$$l(h,x) = [c_1 + c_2(x+1)^2] \exp(x^{-1} a_{\Sigma} uv), \quad x < 0. \quad (3.32)$$

The main observation now is that  $x(u,v)$  is only weakly  $u$ -dependent in regions where  $l(h,x)$  is big. To see this we evaluate  $(\partial x(u,v)/\partial u)_v$ ,

$$\left(\frac{\partial x}{\partial u}\right)_v \approx -\cos \alpha (u^2 - 2 \sin \alpha \cdot uv + v^2)^{-3/2} \\ \times (v^3 - \sin \alpha \cdot uv^2), \quad (3.33)$$

where the spirit of the approximation is the same as that used to get eqs. (3.24') and (3.26'); the exact expression for  $(\partial x/\partial u)_v$  vanishes identically at  $v = v_1 = -\kappa \sec \alpha$ , where  $x = \kappa$ , and for

$v = 0(\kappa)$ ,  $(\partial x/\partial u)_v = 0(\kappa^2)$ . For  $\alpha = 0$ , we have

$$|(\partial x/\partial u)_v| \approx \left[ \left(\frac{u}{v}\right)^2 + 1 \right]^{-3/2}$$

which, even for  $|v| = .4$ , satisfies  $|\partial x/\partial u|_v \approx 5\%$  since  $u \geq 1$  by eq. (3.22); and  $x(1,.4) = .36 \sim 20\kappa$ , for  $\kappa = .018$ , a value larger than is needed for our purposes. So in (3.31) and (3.32) we replace  $x(u,v)$  with  $x(1,v)$ , approximating the curves of constant  $\theta$ , on  $\Sigma$ , by curves of constant  $v$ , for values of  $v$  well away from  $v_1$ , where this result is exact. The results are worked out in Section 4.

Finally we introduce a certain geometrical approximation  $\sum_o$ , to the shape  $\sum$ , which does not enjoy complete generality but which does cover a reasonably wide range of circumstances, those of the present application, in particular. We simply insert constant, suitable chosen limits  $u_1, u_2$ , and  $v_1, v_2$  into the integral on the right side of (3.28) so that

$$\begin{aligned} \frac{1}{2\pi} \int d\Omega g_{\Sigma}(\Omega) \dots &\doteq \frac{1}{2\pi} \int d\Omega g_{\Sigma_o}(\Omega) \dots \\ &\doteq \frac{\cos \alpha}{2\pi} \sum_{i=1}^2 \int_{u_1}^{u_2} du \int_{v_1}^{v_2} dv (u^2 - v^2) [(u^2 - 1)(1 - v^2)]^{-1/2} \\ &\quad \times (u^2 - 2 \sin \alpha \cdot uv + v^2)^{-3/2} \dots, \end{aligned} \quad (3.34)$$

where the sum arises from the fact that the coordinates  $u, v$  only cover half the  $x_o y_o$  - plane, e.g. the  $x \geq 0$  half, and we have assumed that  $x_1 < 0 < x_2$ . If  $0 \notin (x_1, x_2)$ , the sum in (3.34) is replaced by a difference; i.e., a single term, written as  $\int_{u_1}^{u_2} du \int_{v_1}^{v_2} dv \dots$  suffices.

For all of the circumstances of interest to us here we may choose  $u_i$  to correspond to the ellipse passing through the point  $(x_o, y_o) = (x_i, 0)$ ; this gives

$$u_i \equiv \sec \Theta_i = (x_i^2 + D^2)^{1/2} / D \doteq (1 + \xi_i^2 \cos^2 \alpha)^{1/2}. \quad (3.35)$$

The main fact making this choice possible is that in all cases here the upper and lower edges of  $\sum$ , at  $y_o = y_1$  and  $y_2$ , lie between the foci of the coordinate system, at  $P_1 = P_1(0, -bz_1 - D)$  and  $P_2 = P_2(0, -bz_1 + D)$ . In almost all of these the analogous choice

for  $v_i$  is a good enough approximation\*, that  $v_i$  correspond to the hyperbola passing through  $(x_o, y_o) = (0, y_i)$ . Then, by the second of eqs. (3.22), and eqs. (3.18') and (3.21'),

$$v_i = (y_i + bz_1) D^{-1} \doteq \eta_i \cos \alpha + \sin \alpha. \quad (3.36)$$

---

\* The exception is dealt with in Section 5.

Figures 6 and 7 show examples for two of the surfaces from the M113, illustrating all of the above features. The choice of  $\sum_o$  defined by eqs. (3.35) and (3.36) we refer to as the centered approximation.

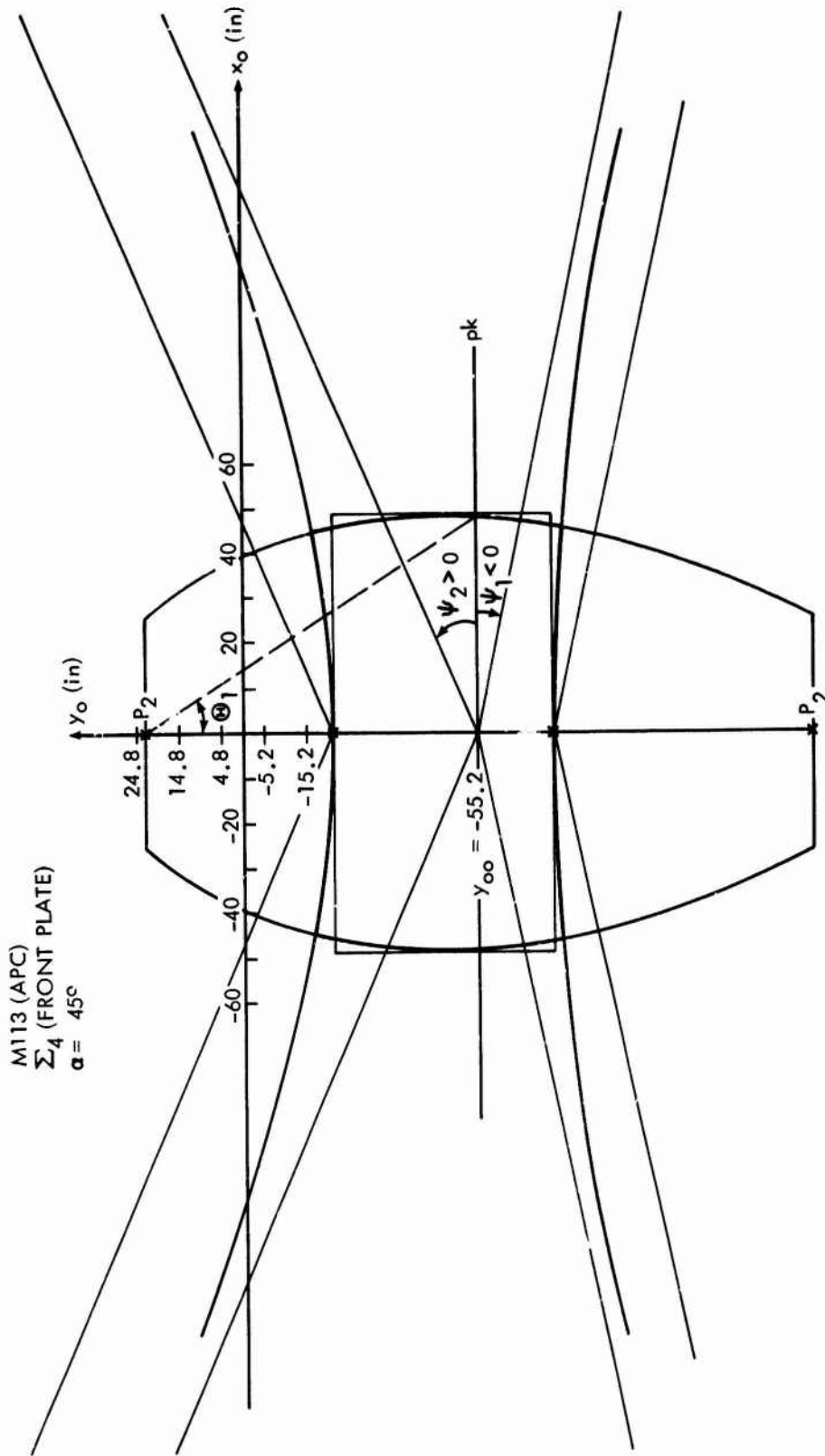


FIG. 6 Centered approximation to the M113 surface  $\Sigma_4$ .  $P_1$  and  $P_2$  are the foci of the elliptical coordinate system and the  $\hat{x}_0$  - and  $\hat{y}_0$  - axes are those of FIG. 5. The curves of constant  $u$  are ellipses and the curves of constant  $v$  are hyperbolas. The point  $x_0 = 0$ ,  $y_0 = y_{00}$  is the mid-point of  $P_1 P_2$ . The ellipse  $u = u_1$  is tangent to the side edges of  $\Sigma_4$ , the hyperbola  $v = v_1$  is tangent to the bottom edge ( $y_0 = y_1$ ), and the hyperbola  $v = v_2$  is tangent to the top edge ( $y_0 = y_2$ ).

M113 (APC)  
 $\Sigma_5$  (LOWER FRONT PLATE)  
 $\alpha = -30^\circ$

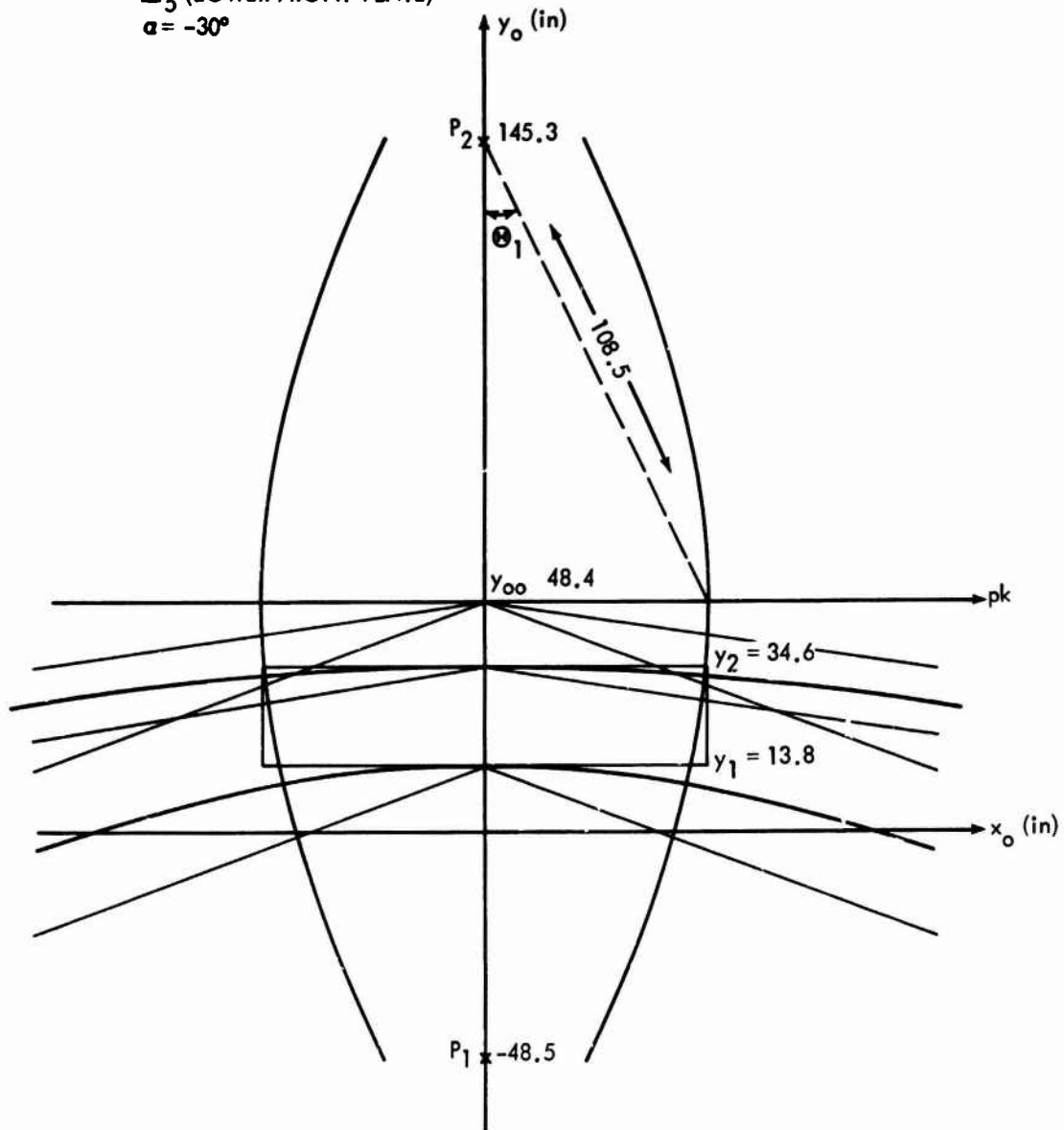


FIG. 7 Same as FIG. 6, but  $\Sigma_5$  rather than  $\Sigma_4$ . Here the ellipse  $u = u_1$  is tangent to the extension of the side edges since the midpoint of  $P_1P_2$  lies above the rectangle ( $y_{ao} > y_1$  and  $y_{ao} > y_2$ ).

4. EXPRESSIONS FOR THE  $E_{\Sigma}$ 's, AND  $R$ .

 A. Class i. Wall Slabs ( $|\alpha|$  Away From  $\pi/2$ ).

It suffices to treat the case  $x_1 = -x_2$  so that  $u_1 = u_2$ ; this is the symmetrical case. For the asymmetrical case,  $u_1 \neq u_2$ , the right side of (3.34) is the average of the two symmetrical results for  $u_1$  and  $u_2$ .

We evaluate (3.34) first with the "... " replaced by 1; the resulting expression gives the solid angle factor for  $\sum_0$  at the detector position, namely  $(2\pi)^{-1}$  times the solid angle subtended at  $D(0,0,z_1)$  by  $\sum_0$ ,

$$w_{\sum_0} \doteq \frac{\cos \alpha}{\pi} \int_1^{u_2} du \int_{v_1}^{v_2} dv (u^2 - v^2) [(u^2 - 1)(1 - v^2)]^{-\frac{1}{2}} \times (u^2 - 2 \sin \alpha \cdot uv + v^2)^{-3/2}. \quad (4.1)$$

Using

$$\rho^{-3} = u^{-2} v^{-1} \sum_{n=1}^{\infty} P_n'(\sin \alpha) \left(\frac{v}{u}\right)^n, \quad |v/u| < 1, \quad (4.2)$$

where

$$\rho = \rho(\sin \alpha) = (u^2 - 2 \sin \alpha \cdot uv + v^2)^{\frac{1}{2}} \quad (4.3)$$

and  $P_n$  is the Legendre polynomial of order  $n$ , the prime denoting differentiation by the argument, we have

$$w_{\sum_0} \doteq \frac{\cos \alpha}{\pi} \int_1^{u_2} du (u^2 - 1)^{-\frac{1}{2}} \int_{v_1}^{v_2} dv (1 - v^2)^{-\frac{1}{2}} [1 - (v/u)^2] v^{-1} \cdot \sum_{n=1}^{\infty} P_n^1(\sin \alpha) (v/u)^n. \quad (4.4)$$

We define

$$I_n = \int_1^{u_2} du u^{-n} (u^2 - 1)^{-\frac{1}{2}} = \int_0^{\Theta_2} d\theta \cos^{n-1} \theta \quad (4.5a)$$

$$J_n = \int_{v_1}^{v_2} dv v^{n-1} (1 - v^2)^{-\frac{1}{2}} = \int_{\Psi_1}^{\Psi_2} d\Psi \sin^{n-1} \Psi, \quad (4.5b)$$

where

$$v_i = \sin \Psi_i, \quad i = 1, 2, \quad (4.6)$$

with  $\Theta_2$  given by eq. (3.35), and interchange orders of integration

and summation to get

$$w_{\Sigma_0} \doteq \frac{\cos \alpha}{\pi} \sum_{n=1}^{\infty} P'_n(\sin \alpha) (I_n J_n - I_{n+2} J_{n+2}) \quad (4.7)$$

$$\doteq \frac{\cos \alpha}{\pi} \sum_{n=0}^{\infty} (2n+1) P_n(\sin \alpha) I_{n+1} J_{n+1}, \quad (4.8)$$

in which we have used the identity  $P'_{k+1}(x) - P'_{k-1}(x) = (2k+1)P_k(x)$ . The key feature of the expansion, eq. (4.8), is that successive terms diminish rapidly owing to eq. (4.5b): the expansion parameters are the  $\psi_i$  because the lowest order term of the Taylor series for  $J_n(\psi_1, \psi_2)$  is  $n^{-1}(\psi_2^n - \psi_1^n)$ . From eqs. (4.5), (4.6) and (4.8) we have

$$w_{\Sigma_0} \doteq \frac{\cos \alpha}{\pi} \left[ \Theta_2 \psi + \frac{3}{2} \sin \alpha \sin \Theta_2 \cdot \psi^2 - \frac{5}{12} (3 \sin^2 \alpha - 1) (\Theta_2 + \sin \Theta_2 \cos \Theta_2) \cdot \psi^3 + \dots \right]_{\psi_1}^{\psi_2} \quad (4.9)$$

The solid angle factor for  $\Sigma$  can be computed exactly for comparison; it is the difference of the solid angle factors for the rectangles  $\Sigma_2$  and  $\Sigma_1$ , having common upper edge at  $y_0 = 0$  and lower edges at (respectively)  $y_0 = y_2$  and  $y_0 = y_1$  (cf Figure 6),

$$w_{\Sigma} = w_{\Sigma_2} - w_{\Sigma_1}, \quad (4.10)$$

where, from eq. (3.12)

$$w_{\Sigma_i} = \frac{1}{\pi} \int_0^{\xi_2} d\xi \int_0^{\eta_i} d\eta (\xi^2 + \eta^2 + 1)^{-3/2} \quad (4.11)$$

$$= \frac{1}{\pi} \tan^{-1} [\eta_1 \xi_2 (1 + \eta_1^2 + \xi_2^2)^{-1/2}] \quad (4.12)$$

For the case shown in Figure 6,  $z_1 = 55.2''$ ,  $y_1 = -72.1''$ ,  $y_2 = -20.8''$ , and  $x_2 = 49.1''$ , which yields

$$w_{\Sigma} = .1016$$

The parameters of  $\Sigma_0$  in the centered approximation are obtained from eqs. (3.21'), (3.35), and (3.36) with  $\alpha = 45^\circ$ ; we find  $D = 78''$ ,  $\Theta_2 = .5620$ ,  $\psi_1 = -.2181$ , and  $\psi_2 = .4556$ , so that eq. (4.9) gives

$$\omega_{\Sigma_0} \pm .0852 + .0203 - .0050 \pm .1005,$$

which agrees with the exact result for  $\omega_{\Sigma}$  to less than 2%.

We apply to  $E_{\Sigma}$  the methods used to obtain eqs. (4.8) and (4.9). With the help of eqs. (3.26'), (3.31) and (3.32) the analogue of eq. (4.4) is

$$\begin{aligned} E_{\Sigma_0} &\pm \frac{\cos \alpha}{\pi} \int_1^{u_2} du (u^2 - 1)^{-\frac{1}{2}} \int_{v_1}^{v_2} dv (1 - v^2)^{-\frac{1}{2}} [1 - (v/u)^2] v^{-1} \\ &[\theta(-v) c x^{-1} \exp(-\kappa x^{-1}) + \theta(v) (c_1 + c_2 (x+1)^2)] \\ &\cdot \exp(-A_{\Sigma} u) \sum_{n=1}^{\infty} P'_n(\sin \alpha) (v/u)^n, \end{aligned} \quad (4.13)$$

where  $\theta(v)$  denotes the unit step function, which vanishes for  $v < 0$ , and

$$A_{\Sigma} \equiv -a_{\Sigma} v x^{-1} \quad (4.14)$$

In the approximation discussed near the end of Section 3,  $x(u, v) \approx x(1, v)$ , so that also  $A_{\Sigma}(u, v) \approx A_{\Sigma}(1, v)$ . In parallel with eqs. (4.5) we define

$$I_n(v) = \int_1^{u_2} du u^{-n} (u^2 - 1)^{-\frac{1}{2}} \exp(-A_{\Sigma}(1, v) u) \quad (4.15a)$$

$$J_n = \int_{v_1}^{v_2} dv v^{n-1} (1 - v^2)^{-\frac{1}{2}} \ell_1(v), \quad (4.15b)$$

where  $\ell_1(v)$  stands for the square bracketed expression in (4.13) with  $x = x(1, v)$ . We write  $I_n(v)$  as a Taylor series in  $v$ ,

$$I_n(v) = \sum_{m=0}^{\infty} \frac{v^m}{m!} I_n^{(m)}(0), \quad (4.16)$$

substitute this with eqs. (4.15) into (4.13) and rearrange the summations to get the analogue of eq. (4.8),

$$E_{\Sigma_0} \pm \frac{\cos \alpha}{\pi} \sum_{r=0}^{\infty} J_{r+1} \sum_{m=0}^r (2n+1) P_n(\sin \alpha) \frac{I_{n+1}^{(r-n)}(0)}{(r-n)!} \quad (4.17)$$

Keeping only the first two terms, we obtain an approximation whose spirit is only a little rougher than that of eq (4.9),

$$E_{\Sigma} \approx \frac{\cos \alpha}{\pi} \left[ I_0^{(0)}(0) J_1 + (I_1^{(1)}(0) + 3 \sin \alpha I_2^{(0)}(0)) J_2 \right]. \quad (4.18)$$

The analogous approximation to  $w_{\Sigma}$  for the example of Figure 6, just discussed, agrees with  $w_{\Sigma}$  to a few percent (1.4%). We note that

$$I_n^{(0)}(0) = I_n(0) \quad (4.19a)$$

$$\begin{aligned} I_n^{(1)}(0) &= -\left. \frac{dA_{\Sigma}}{dv} \right|_0 \cdot I_{n-1}(0) \\ &= a_{\Sigma} \tan \alpha \cdot I_{n-1}(0), \end{aligned} \quad (4.19b)$$

so eq. (4.18) may be rewritten as

$$E_{\Sigma_0} \approx \frac{\cos \alpha}{\pi} \left[ I_1(0) J_1 + (3 \sin \alpha \cdot I_2(0) + a_{\Sigma} \tan \alpha \cdot I_0(0)) J_2 \right] \quad (4.20)$$

If  $\alpha = 0$ , this has an especially simple form;

$$E_{\Sigma_0} \approx \frac{1}{\pi} I_1(0) J_1, \quad \alpha = 0. \quad (4.21)$$

To complete the scheme we develop expressions for  $J_1$  and  $J_2$  for use in eq. (4.20). The  $I_n(0)$  present no difficulty because  $I_1(0)$  is the Sievert integral; changing variables in eq. (4.15a),

$$I_1(0) = \int_0^{\Theta_2} d\theta \exp(-A_{\Sigma}(1,0) \sec \theta) = S(A_{\Sigma}(1,0), \Theta_2), \quad (4.22a)$$

which is tabulated;<sup>5</sup> we note from eqs. (3.26') and (4.4) that

$$A_{\Sigma}(1,0) = a_{\Sigma} \sec \alpha. \quad (4.22b)$$

$I_0(0)$  is found by differentiating (4.22a),

$$\begin{aligned} -\frac{\partial S(A, \Theta_2)}{\partial A} &= \int_0^{\Theta_2} d\theta \sec \theta \exp(-A \sec \theta) \\ &= \int_1^{u_2} du (u^2 - 1)^{-\frac{1}{2}} \exp(-Au), \end{aligned} \quad (4.23)$$

and this can be determined from the tabulation of  $S$  to adequate accuracy by numerical differentiation. Finally by examining the expansions of  $I_n(0)$  in powers of  $\Theta_2$  for  $n = 0$  and  $n = 2$  we can produce the approximation,

$$I_2(0) \approx -I_0(0) + 2 I_1(0) + \frac{1}{20} \Theta_2^5 \exp(-a_{\Sigma} \sec \alpha). \quad (4.24)$$

The coefficients of the expansions of both sides agree up to and including the  $\Theta_2^5$  - term.

Turning to  $J_1$  and  $J_2$ , we treat the direct and skyshine parts separately, writing

$$J_n = J_n^{(d)} + J_n^{(s)} \quad (4.25)$$

where the integration in  $J_n^{(d)}$  extends over the range of values of  $v$  for which  $x > 0$ , while in  $J_n^{(s)}$ ,  $x < 0$  always holds. In the integrand of the direct contribution we extract a peak factor,

$$-(v \cos \alpha)^{-1} \exp [+ \kappa (v \cos \alpha)^{-1}],$$

and expand the rest in powers of  $v$ . Integrating term by term produces a series of exponential integrals,  $E_k(\zeta_i)$ , having small arguments when  $\kappa \ll 1$

$$E_k(\zeta_i) = \int_{\zeta_i}^{\infty} d\zeta \zeta^{-k} e^{-\zeta}$$

where

$$\zeta_i = \kappa \sec \alpha \cdot (-v_i)^{-1}, \quad (4.26)$$

in which  $v_i$  means  $v_1$  or  $v_2$ . Using

$$E_2(\zeta_i) = \zeta_i^{-1} \exp(-\zeta_i) - E_1(\zeta_i),$$

we find, for  $v_1 < 0 < v_2$ ,

$$J_1^{(d)} \approx c \sec \alpha \left\{ E_1(\zeta_1) - \left[ e^{-\zeta} \sin \alpha (1 - \kappa (\tan \alpha + \frac{1}{2} \cot \alpha)) v \right]_{v_1}^0 \right\}, \quad (4.27a)$$

and for  $v_1 < v_2 < 0$ ,

$$J_1^{(d)} \approx c \sec \alpha \left\{ \left[ E_1(\zeta) - e^{-\zeta} \sin \alpha (1 - \kappa (\tan \alpha + \frac{1}{2} \cot \alpha)) v \right]_{v_1}^{v_2} \right\}; \quad (4.27b)$$

also, for  $v_1 < 0 < v_2$ ,

$$J_2^{(d)} \approx c \sec \alpha \left\{ -\kappa \sec \alpha E_1(\zeta) + \left[ e^{-\zeta} (1 - \kappa \tan \alpha) v \right]_{v_1}^0 \right\} \quad (4.28a)$$

and, for  $v_1 < v_2 < 0$ ,

$$J_2^{(d)} \approx c \sec \alpha \left\{ \left[ -\kappa \sec \alpha E_1(\zeta_1) + e^{-\zeta} (1 - \kappa \tan \alpha) v \right]_{v_1}^{v_2} \right\}. \quad (4.28b)$$

For the skyshine part we expand the integral and integrate: for

$$v_1 < 0 < v_2,$$

$$J_1^{(s)} \approx (c_1 + c_2) v \Big|_0^{v_2}, \quad J_2^{(s)} \approx 0, \quad (4.29)$$

and for  $v_1 < v_2 < 0$ ,

$$J_1^{(s)} = 0, \quad J_2^{(s)} = 0. \quad (4.30)$$

In eqs. (4.27) through (4.29) we have dropped terms of order  $\kappa^2$  and, in comparison with  $E_1(\zeta_1)$ , we have neglected terms of order  $v_1^2$  and  $v_2^2$ .

### B. Class ii. Floor and Ceiling Slabs ( $\alpha = \pm \frac{\pi}{2}$ ).

The method we use in these cases is described in Reference 2, so we mainly just give the results of using the source indicated in eq. (2.20).

#### Floor ( $\alpha = -\pi/2$ ).

We suppose the detector position to be above the center of a rectangular floor  $\Sigma$ .  $\Sigma_0$  is a shape compounded of a centered circle of radius  $r$ , and two equal, symmetrically placed, annular sectors having radii  $r_1$  and  $r_2 > r_1$ , concentric and with angle  $\psi_0$ .  $r_1$  is determined by the requirement that the solid angle factor subtended at D by the circle equal that of a concentric square having side equal to the width of  $\Sigma$ , which is  $\omega_{\Sigma}'$ ;  $r_2$  is determined the same way, with the side of the square equal to the length of  $\Sigma$ , the solid angle factor being  $\omega_{\Sigma}''$ , and  $\psi_0$  is fixed by requiring  $\omega_{\Sigma_0} = \omega_{\Sigma}$ .

#### Ceiling ( $\alpha = +\pi/2$ ).

Since the skyshine is only about 10% of the total, a coarser approximation will do. We let  $\Sigma_0$  be a circle whose radius,  $r_1$ , is so chosen that  $\omega_{\Sigma_0} = \omega_{\Sigma}$ . This may be regarded as a special case of the surface used above to represent the floor, one in which  $r_2 = r_1$ .

If  $\Sigma$  already is a square,  $\omega_{\Sigma}' = \omega_{\Sigma}'' = \omega_{\Sigma}$  and  $r_2 = r_1$  automatically hold, and  $\Sigma_0$  becomes a circle.

With

$$r_i \equiv z_1 \lambda_i \equiv z_1 \tan \theta_i, \quad i = 1, 2, \quad (4.31)$$

we have

$$\sec \theta_i = (1 - w_{\Sigma(i)})^{-1}, \quad i = 1, 2, \quad (4.32)$$

$$\varphi_0/\pi = (w_{\Sigma}'' - w_{\Sigma}')^{-1} (w_{\Sigma} - w_{\Sigma}'). \quad (4.33)$$

Expressions for the  $w_{\Sigma(i)}$  may be produced easily from eq. (4.12). From the form (see eqs. (2.3), (2.20), (3.14), (3.15), (3.12)),

$$E_{\Sigma} = \frac{1}{2\pi} \int_{\xi_1}^{\xi_2} \int_{\eta_1}^{\eta_2} d\xi d\eta (\xi^2 + \eta^2 + 1)^{-1} c \exp \left[ -A(\xi^2 + \eta^2 + 1)^{\frac{1}{2}} \right], \quad \alpha = -\pi/2, \quad (4.34)$$

where

$$A \equiv a_{\Sigma} + \kappa, \quad (4.35)$$

we convert to integration over  $\lambda$ , which gives

$$E_{\Sigma_0} = c \int_0^{\lambda_1} d\lambda (\lambda^2 + 1)^{-1} \exp \left[ -A(\lambda^2 + 1)^{\frac{1}{2}} \right] + (\varphi_0/\pi) c \int_{\lambda_1}^{\lambda_2} d\lambda (\lambda^2 + 1)^{-\frac{1}{2}} \exp \left[ -A(\lambda^2 + 1)^{\frac{1}{2}} \right] \quad (4.36)$$

$$= E_1(A) - [1 - (\varphi_0/\pi)] E_1(A \sec \theta_1) - (\varphi_1/\pi) E_1(A \sec \theta_2); \quad (4.37)$$

and from the form,

$$E_{\Sigma} = \frac{1}{2\pi} \int_{\xi_1}^{\xi_2} \int_{\eta_1}^{\eta_2} d\xi d\eta (\xi^2 + \eta^2 + 1)^{-\frac{1}{2}} \left\{ c_1 + c_2 [1 - (\xi^2 + \eta^2 + 1)^{-\frac{1}{2}}]^2 \right\} \\ \times \exp \left[ -A_{\Sigma} (\xi^2 + \eta^2 + 1)^{\frac{1}{2}} \right], \quad \alpha = +\pi/2, \quad (4.38)$$

derived the same way as eq. (4.34), we find

$$E_{\Sigma_0} = \left[ -c_1 \left\{ y^{-1} \exp(-a_{\Sigma} y) + a_{\Sigma} E_1(a_{\Sigma} y) \right\} - c_2 \left\{ y^{-1} \exp(-a_{\Sigma} y) \right. \right. \\ \times \left[ (1 + a_{\Sigma} + \frac{1}{6} a_{\Sigma}^2) - (1 + \frac{1}{6} a_{\Sigma}) y^{-1} + \frac{1}{3} y^{-2} \right] \\ \left. \left. + (1 + a_{\Sigma} + \frac{1}{6} a_{\Sigma}^2) a_{\Sigma} E_1(a_{\Sigma} y) \right\} \right]_{y=1}^{y=\sec \theta_1}. \quad (4.39)$$

C. Approximation of the Reduction Factor.

The general approximate form for R is

$$R \approx R_0 \equiv \sum B_{oi} E_{\Sigma_{oi}} \quad (4.40)$$

In most cases the  $\Sigma_0$  choices described in this section and the last, e.g. the centered approximation, are adequate approximations to  $\Sigma$ , giving results which are off only by a few percent. But Figure 8 shows an example in which the centered approximation to a rectangular representation of a slab  $\Sigma$  entails a sizable distortion. Here we may retain eq. (3.35), but change the values of the  $v_i$  so that cancellation in  $E_{\Sigma_0}$ , of errors from portions of  $\Sigma$  near the middle of and exterior to  $\Sigma_0$ , with errors at the corners, in  $\Sigma_0$  but exterior to  $\Sigma$  can occur. This results in  $v_i$  slightly smaller in absolute value than values stemming from eq. (3.36). The best values of  $v_i$ , for example, can be estimated by equating the ratio of the areas of the (right) "triangles" with centroids A and B in the figure to the values of the integrand of  $E_{\Sigma}$  at these two points. This condition gives the base of triangle A, which determines  $v_1$ . This is the method we have used to improve the choice of  $\Sigma_0$  for the long side walls of the M113 in the next section. For identification purposes we call this the "reshaping approximation".

# 5. "WALLS ONLY" MODEL OF THE M113.

In the phrase "walls only" we intend that the floor and ceiling surfaces also be included. Figure 8 shows the indexing convention for the surfaces of the M113. To achieve the idealization implied by the contents of the figure we have identified the inner framing plates with the hull walls, ignored a variety of surface bumps and irregularities, and represented the sides of the vehicle as flat. Figure 9 is a sketch of the vehicle cross section as seen from the rear, together with some aspects of the idealization implied by our procedure.

We choose the detector position to be 30 inches above the floor, or 45.9 inches above the ground, so that  $h_{eff} = (36 + 45.9)$  in = 6.8 ft, and  $\kappa = .0158$ . The detector is centered, equidistant from the two sides, and also more or less is centered with respect to front and rear, beneath the cupola opening. In Table 1 we list the data for each of the  $\Sigma_i$ , much of which partially is determined by choice of detector position. In Table 2 we list the computed values of the\*  $E_{\Sigma_{oi}}$ , the buildup factors, and the  $r_{\Sigma_{oi}}$ .

By a process of inspired scrutiny involving eq. (3.30) we have estimated crudely a 1.05 refraction correction; also, we assume a 10% albedo, so that from eq. (4.40) and Table 2,

$$R = 1.155R = .6521 \quad (5.1)$$

$$P = R^{-1} = 1.53 \quad (5.2)$$

---

\* For  $\Sigma_6(\Sigma_7)$  we have computed  $E_{\Sigma_0}$  for the detector position above (below) the center of  $\Sigma_6(\Sigma_7)$  for simplicity. This is almost exactly the case for  $\Sigma_6$  so the error here is small; it is greater for  $\Sigma_7$ , but this is tolerable because the total skyshine contribution is small, anyway.

Also we have divided  $\Sigma_1(=\Sigma_2)$  into  $\Sigma_1^\uparrow + \Sigma_1^\downarrow$ , where  $\Sigma_1^\uparrow$  is the portion of  $\Sigma_1$  above the horizontal plane containing the detector and  $\Sigma_1^\downarrow$  the portion below. For  $\Sigma_1^\uparrow$  we have taken  $u_1 \neq u_2$  to take account of the shape irregularity. By the remark in paragraph 1 of Section 4,  $E_{\Sigma_1^\uparrow} = \frac{1}{2}(E_{\Sigma_{1L}^\uparrow} + E_{\Sigma_{1R}^\uparrow})$  where L and R refer to the left ( $u = u_1$ ) and right ( $u = u_2$ ) portions of  $\Sigma_1^\uparrow$ .

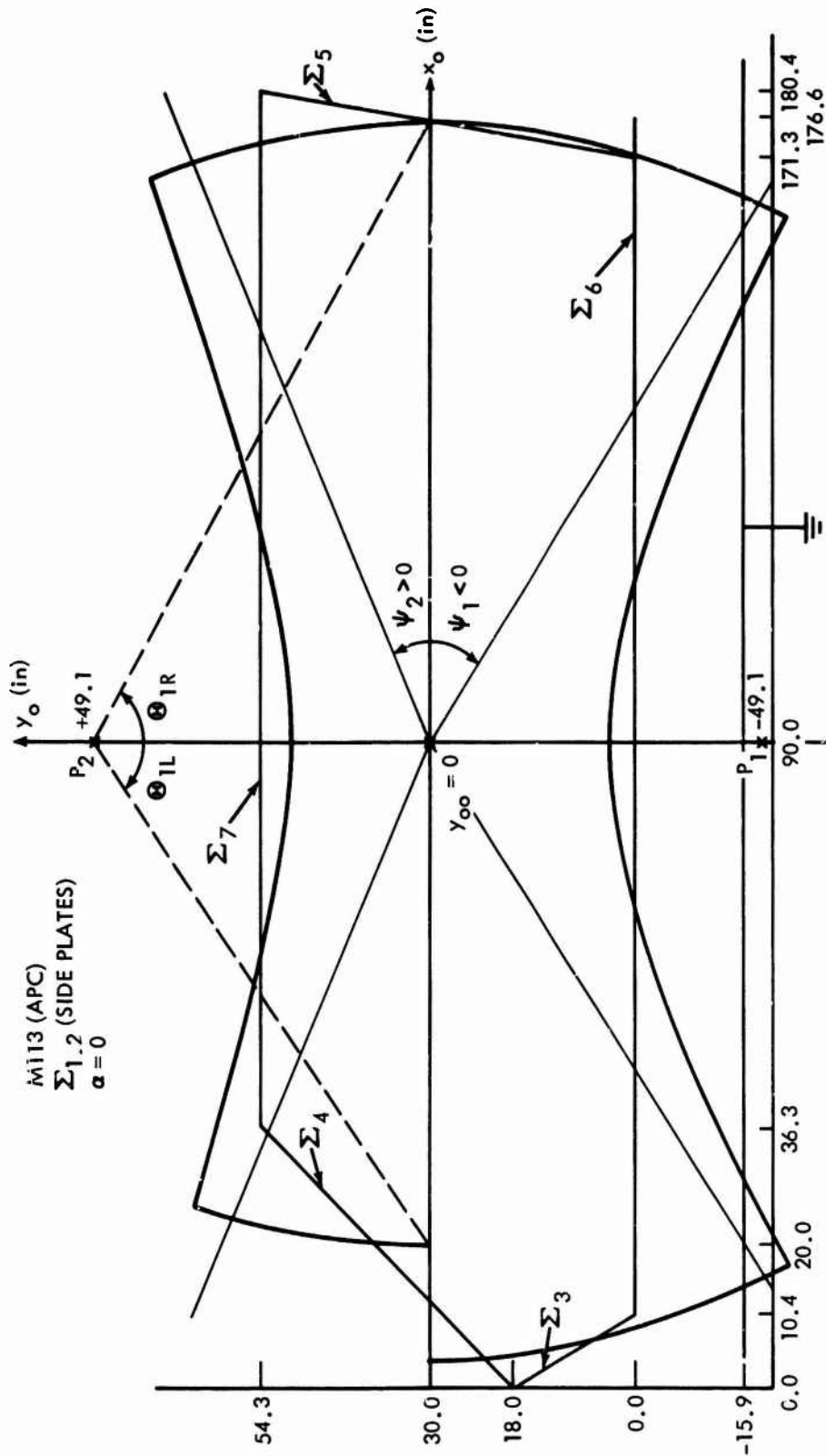


FIG. 8 "Reshaping approximations" to the rectangles used to represent the M113 surface  $\Sigma_1 = \Sigma_1^{\downarrow} + \Sigma_1^{\uparrow}$ .  $u_1$  and  $u_2$  are chosen the same way as in the centered approximation (FIG. 6 and 7), but  $v_1$  and  $v_2$  are chosen differently, as described in the text. The necessity arises from the fact that the dimensions of the slab are big, in relation to  $P_1 P_2 = 2D = 2z_1 \sec \alpha$ . The illustration also shows the indexing convention for the remaining surfaces of the M113.

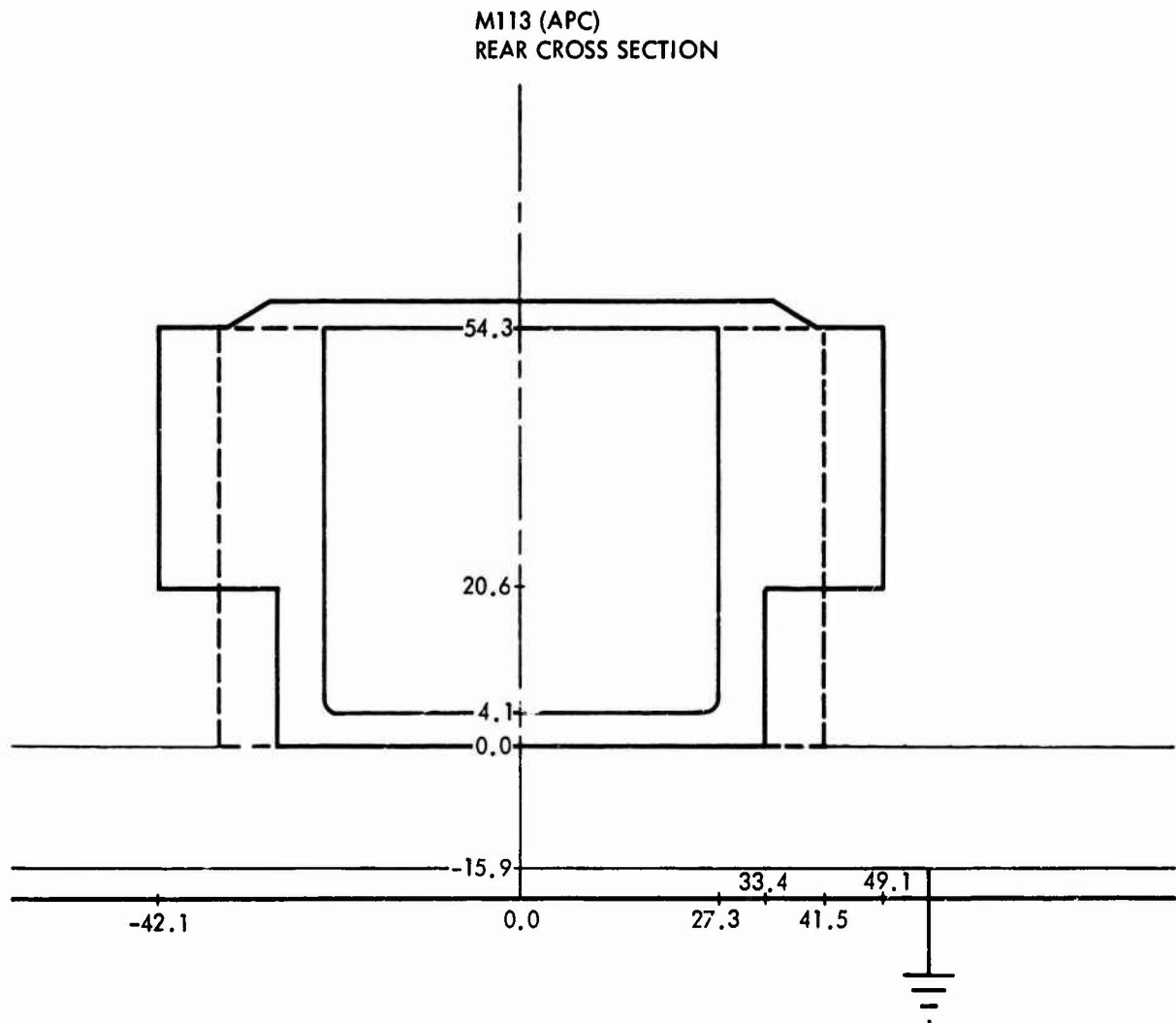


FIG. 9 M113 vehicle cross section, from the rear, showing the double wall structure.  
The dotted lines trace out the rectangular cross section for the "walls only" model.

	$\Sigma_1^{\downarrow} = \Sigma_2^{\downarrow}$	$\Sigma_1^{\uparrow} = \Sigma_2^{\uparrow}$	$\Sigma_3$	$\Sigma_4$	$\Sigma_5$	$\Sigma_6$	$\Sigma_7$
$\alpha$	0	0	$-9.854^\circ$	$+45^\circ$	$-30^\circ$	$-90^\circ$	$+90^\circ$
t	1.875	1.875	1.500	1.500	1.500	1.500	1.625
$z_1$	49.1	49.1	84.7	55.2	83.9	30.0	24.5
$-x_1, x_2$	86.0	70.0, 86.0	49.1	49.1	49.1	83.1	72.1
$y_1$	-30.0	0	-15.7	-72.1	+13.8	-49.1	-49.1
$y_2$	0	+24.3	+59.4	-20.8	+34.6	+49.1	+49.1
$u_1, u_2$	2.02	1.74, 2.02	1.1516	1.1816	1.1211	---	---
$v_1$	-.520	0	-.3539	-.2164	-.3575	---	---
$v_2$	0	+.420	+.2871	+.4407	-.1428	---	---
$\theta_1, \theta_2$	1.0495	.9580, 1.0495	.5189	.5620	.4691	---	---
$\psi_1$	-.547	0	-.3618	-.218	-.3656	---	---
$\psi_2$	0	+.433	+.2912	+.456	-.1433	---	---

Table 1. Slab data for computing the M113 protection factor in the "walls only" model. Entries for dimensioned quantities, in rows 2 - 6, all are expressed in inches; entries in the last three rows, 10 - 12, are in radians.

	$\Sigma_1^{\downarrow} = \Sigma_2^{\downarrow}$	$\Sigma_1^{\uparrow} = \Sigma_2^{\uparrow}$	$\Sigma_3$	$\Sigma_4$	$\Sigma_5$	$\Sigma_6$	$\Sigma_7$
$E_{\Sigma_0}$	.0873	.0114	.0516	.0272	.0101	.0766	.0144
$B_0$	1.543	1.548	1.438	1.438	1.438	1.438	1.475
$r_{\Sigma_0}$	.1351	.0176	.0742	.0391	.0145	.1102	.0212

Table 2. Attenuation, buildup, and reduction factors for the surfaces of the "walls only" model of the M113.

From eq. (2.6), using  $L(6.8 \text{ ft.}) = .86$  we find

$$S = 1.32, \quad (5.3)$$

which is considerably smaller than the average value reported in Reference 1,

$$S = 3.41, \quad (5.4)$$

where variations of detector position did not involve deviations of more than 10 to 15%.

The results of the simplified model of Section 6 support the hypothesis that the discrepancy between (5.3) and (5.4) is due to neglect of shielding effects from material present additionally to that which resides in the vehicle's surfaces.

## 6. PARALLELIPIPED MODEL

The combat loaded weight of the M113 is 24,000 pounds<sup>7</sup> while the weight of material assumed for the calculation of Section 5 is

$$W_{\text{shell}} = \rho_{\text{Al}} \sum_i A_i t_i \quad (6.1)$$

where  $\rho_{\text{Al}} = 2.72 \rho_{\text{H}_2\text{O}} = 2.72 \times 62.4 \text{ \#/ft}^3 = 117 \text{ \#/ft}^3$ , the density

of Aluminum,  $A_i$  is the area of  $\Sigma_i$ , and  $t_i$  its thickness. The volume factor in eq. (6.1) for the M113 is

$$\sum_i A_i t_i = 57.96 \text{ ft}^3 \quad (\text{M113}) \quad (6.2)$$

so that  $W_{\text{shell}} = 9838 \text{ \#}$ , which is only 41% of the combat loaded weight given above. In light of this, the disagreement between (5.3) and (5.4) is hardly unreasonable. We proceed to a simplified model and calculation for the M113, based on the information and experience gained so far.

We idealize the vehicle by a closed shell of uniform thickness and (possibly heterogeneous) composition. The attenuation constant for the unscattered component of the radiation due to atoms of species  $k$  is

$$\mu_k = n_k \sigma_k \quad (6.3)$$

when  $n_k$  is the number density for atoms of species  $k$  and  $\sigma_k$  the corresponding  $\gamma$ -ray cross section. We define an effective<sup>k</sup> thickness,

$$t_{\text{eff}} = W/(\rho_{\text{Al}} A), \quad (6.4)$$

where  $W$  is the total weight of the fully loaded vehicle and  $A$  is the surface area of the shell used to model it. Then the unscattered component penetrating the wall is proportional to

$\exp(-\bar{\mu} t_{\text{eff}})$ , where

where

$$\bar{\mu} = \left( \sum_k \mu_k t_k \right) t_{\text{eff}}^{-1} \quad (6.5)$$

$$= \mu_{\text{Al}} \sum_k f_k g_k, \quad (6.6)$$

with

$$f_k = W_k/W \quad (6.7a)$$

$$g_k = (Z/A)_{\text{Al}}^{-1} (Z/A)_k, \quad (6.7b)$$

wherein  $W_k$  is the weight of material of species  $k$ ,  $Z$  is the number of electrons on the indicated atom and  $A$  the corresponding atomic weight.

The composition of the M113 was not available to us, so to estimate  $\bar{\mu}$  we have modeled the material breakdown as follows:

Aluminum	10,000 <sup>#</sup>
Fuel	2,000 <sup>#</sup>
13/160 <sup>#</sup> dummies	2,100 <sup>#</sup>
Other	9,900 <sup>#</sup>
Total	<u>24,000<sup>#</sup></u>

The "other" entry includes the motor, hatches, gun, treads, etc., of which we assume about 2/3 to be iron, with specific gravity = 7.86, and the rest aluminum. The results, together with the values assumed or computed for the  $f_k$  and the  $g_k$ , are given in Table 3. The result is

$$\bar{\mu}/\mu_{Al} = 1.02$$

so we just take

$$\bar{\mu} = \mu_{Al} \quad (6.8)$$

as the attenuation constant for the shell wall, and

$$t = t_{eff} \quad (6.9)$$

as its thickness.

In the present model we have concentrated all the material present into the shell, disregarding effects of the actual, quite complicated, distribution of matter. In the same spirit we choose the simplest shape to work with, a parallelepiped having a square vertical cross section of side  $L$ , and a height  $H$  equal to that of the actual vehicle. We fix  $L$  by equating the surface area of the square parallelepiped to that of the best rectangular/parallelepiped representation of the vehicle's actual shape, which we estimate from the Master Diagram<sup>6</sup>:

$$\left. \begin{array}{l} \text{length} = 162.3 \text{ in.} \\ \text{width} = 83.0 \text{ in.} \\ H = 54.3 \text{ in.} \end{array} \right\} \quad (M113) \quad (6.10)$$

This gives  $A = 372 \text{ ft}^2$ ,  $t_{eff} = 4.56 \text{ in.}$ ,  $L = 118.1 \text{ in.}$

Species	Sp. Grav.	$W_k$ (#)	$f_k$	$g_k$
Aluminum	2.7	13,400	.56	1.00
Iron	7.86	6,500	.27	.95
Dummies	1.8	2,100	.09	1.10
Fuel	1.0	2,000	.08	1.23

Table 3. Estimated material breakdown for the M113.

For all the walls  $\alpha = 0$ , so we may use eq. (4.21). We compute just the direct contribution  $R^{(d)}$  to  $R$ , correcting for skyshine and albedo together by multiplying by 1.1 (skyshine) x 1.1 (albedo) = 1.21;

$$R = 1.21 R^{(d)}, \quad (6.11)$$

where

$$R^{(d)} = B_o(t_{eff})(E_f + \sum_{i=1}^4 E_{w_i}^{(d)}), \quad (6.12)$$

with suffixes  $f$  and  $w$  referring to floor and wall, with  $B_o(t_{eff})$  given by eq. (2.27), and with

$$E_{w_i}^{(d)} \doteq \frac{1}{\pi} (J_1^{(d)} I_1(o))_i. \quad (6.13)$$

We place the detector on the vertical axis through the center of the square so all the  $E_{w_i}^{(d)}$  are equal; this also makes  $\Sigma_o$  a circle for the floor slab. For the wall we assume the centered approximation, eqs. (3.35) and (3.36). We assume the detector to be at a height  $h_f$  above the floor. Collecting all the necessary formulas specialized to this case we have:

$$P^{-1} = R = 1.21 R^{(d)} \quad (6.14a)$$

$$R^{(d)} = B_o(t_{eff}) \cdot (E_f + 4 E_w^{(d)}) \quad (6.14b)$$

$$B_o(t_{eff}) = 1 + .292 t_{eff} \text{ (in)} \quad (6.14c)$$

$$E_f = c(E_1(A) - E_1(A \sec \theta_1)) \quad (6.14d)$$

$$A = a_\Sigma + \kappa, \quad a_\Sigma = \mu_A t_{\text{eff}} \quad (6.14e)$$

$$\kappa = .002314(\text{ft})^{-1} \cdot (h_f + 15.9 \text{ in} + 3 \text{ ft}), \quad (6.14f)$$

$$\sec \theta_1 = (1 - w_f)^{-1} \quad (6.14g)$$

$$w_f = 2\pi^{-1} \tan^{-1} \xi_{2f}^2 (1 + 2\xi_{2f}^2)^{-\frac{1}{2}} \quad (6.14h)$$

$$\xi_{2f} = L/(2h_f) \quad (6.14i)$$

$$E_w^{(d)} = \pi^{-1} J_1^{(d)} I_1(0) \quad (6.14j)$$

$$I_1(0) = S(a_\Sigma, \theta_2 = 45^\circ) \quad (6.14k)$$

$$J_1 = cE_1(\zeta_1), \quad c = .2034 + .783\kappa, \quad (6.14l)$$

$$\zeta_1 = -\kappa v_1^{-1} \quad (6.14m)$$

$$v_1 = -2h_f/L \quad (6.14n)$$

The sources, respectively, are eqs. (2.5) and (6.10), (6.11), (2.27), (4.37), (4.35) and (3.29), (2.8) and (2.22) (with  $h = h_f + 15.9$  in. replaced by  $h + \Delta h$ ,  $\Delta h = 3$  ft ground roughness, and where  $15.9$  is the height above ground of the floor slab), (4.32), (4.10) and (4.12), (3.11), (4.21), (3.35) and (4.22), (4.27b) and (2.25), (4.26), and (3.36).

We have not included a refraction correction here as we did in Section 5 (cf. eq. (5.1) since the magnitude of the effect, which is the "softening" of the unscattered attenuation at large  $\theta_c$ , is well overshadowed by that arising from the irregularities of the actual mass distribution. Also, the neglect of shape effects implied by the assumption of square vertical cross section renders such a consideration doubtful. Finally, a 5% correction can be absorbed by modelling the vehicle so that  $L$  is a little smaller; for the M113, changed, slightly smaller, dimensions of the "best" rectangular parallelepiped representation of the vehicle's actual shape involve estimates as reasonable as those given in (6.10). A similar remark holds for the P7.

We see from eqs. (6.14) that  $P$  and  $\mathcal{R}$  are fixed as functions of  $h_f$  when  $t_{\text{eff}}$  and  $L$  are given, as, e.g., in (6.10) for the M113. The answers, for two choices of detector height,  $h_f$ , above the floor of the vehicle, are

$$\left. \begin{aligned} R &= R(h_f) = .2734 & h_f &= 25.9 \text{ in.} \\ &= .2544 & h_f &= 45.9 \text{ in.} \end{aligned} \right\} \quad (6.15)$$

so that

$$\left. \begin{aligned} P(25.9 \text{ in.}) &= 3.66 \\ P(45.9 \text{ in.}) &= 3.93 \end{aligned} \right\} \quad (6.16)$$

and, by eq. (2.6),  $S = .86 P$ ,

$$\left. \begin{aligned} S(25.9 \text{ in.}) &= 3.15 \\ S(45.9 \text{ in.}) &= 3.38 \end{aligned} \right\} \quad (6.17)$$

From Reference 1, for positions 18 in from the (two) side walls, experiment gives

$$\left. \begin{aligned} S(47.25 \text{ in.}) &= 3.59, 3.07 \\ S(49.25 \text{ in.}) &= 3.76, 3.24 \end{aligned} \right\} \quad (6.18a)$$

which gives an idea of the experimental spread arising from the matter distribution. The average of these values of  $S$  is 3.42, suggesting

$$S(48.25 \text{ in.}) = 3.4 \pm .3. \quad (6.18b)$$

Experiment also gives, 27 in. from the left wall,

$$S(31 \text{ in.}) = 3.61, \quad (6.18c)$$

a value within the indeterminacy\* indicated in (6.18b). Comparing these values with our computed result, eqs. (6.17), we see that the agreement is good. ("Too good", of course; a 15% albedo weakens the agreement by 5%.)

Turning to the marine corps LVTP7, we take<sup>8</sup>

$$\left. \begin{aligned} \text{length} &= 293.0 \text{ in.} \\ \text{width} &= 103.6 \text{ in.} \\ H &= 78.0 \text{ in.} \end{aligned} \right\} \quad (P7) \quad (6.19)$$

which gives  $A = 851 \text{ ft}^2$ ,  $t_{\text{eff}} = 4.33 \text{ in.}$ ,  $L = 181.6 \text{ in.}$ , and

$$\left. \begin{aligned} h_f = 25.9 \text{ in.} : R &= .2562, P = 3.90, S = 3.35 \\ h_f = 45.9 \text{ in.} : R &= .2471, P = 4.05, S = 3.48 \end{aligned} \right\} \quad (6.20)$$

---

\*As distinguished from "uncertainty!"

We adopt for the residual radiation transmission factor (RRTF) the following:

$$\begin{aligned} \text{M113: } \text{RRTF} &= S^{-1} = (3.4)^{-1} = .29 \text{ experiment} \\ S^{-1} &= (3.5)^{-1} = .29 \text{ theory} \end{aligned} \quad (6.21)$$

$$\text{P7: } \text{RRTF} = S^{-1} = (3.7)^{-1} = .27 \text{ theory.} \quad (6.22)$$

Accordingly we recommend the adoption of the value

$$\text{RRTF} = .3 \quad (\text{LVTP7}). \quad (6.23)$$

## 7. DISUCSSION

The RRTF for both vehicles is 0.3 despite the fact that the P7 weighs more than twice as much as the M113. Note, however, that  $t_{\text{eff}}^{\text{M113}} = 4.56$  in and  $t_{\text{eff}}^{\text{P7}} = 4.33$  in, so the model wall thickness, which is a principal determining factor, is nearly the same for the two cases. This reflects the fact that the extra weight is a size effect,  $L^{\text{M113}} = 118.1$  in and  $H^{\text{M113}} = 54.3$  in, being somewhat less than the corresponding values for the P7,  $L^{\text{P7}} = 181.6$  in. and  $H^{\text{P7}} = 78.0$  in. Moreover, since the floors of both vehicles are 15.9 in. from the ground, equal values of  $h_f$  and  $h$  mutually imply each other; and since  $H^{\text{P7}} > H^{\text{M113}}$  proportionately more of the total mass of the P7 than of the M113 lies above the horizontal plane through the detector position. This means that much of the P7's added mass is positioned so that it protects only against the relatively ineffectual skyshine "threat" ( $\theta > \pi/2$ ).

## 8. SUMMARY AND CONCLUDING REMARK

We have computed the residual radiation transmission factor for the Marine Corps LVTP7 to be 0.3, based on a model of the mass distribution whose results for application to a similar vehicle, the army APC M113, agree with experiment.

We have accomplished this by developing a new analytical approximation scheme for transmission through a rectangular (wall) slab. The scheme has a wide range of applications, to the case where the plane of the slab is not too close to the horizontal: specifically,  $|\alpha|$  not too close to  $\cos^{-1}\kappa$ . The method employs the approximation that the conics determined by the intersection of the cones of constant  $\theta$  with the plane of the slab coincide with the hyperbolic arcs of a natural elliptical coordinate system in the plane of  $\Sigma$ . The coordinate system is chosen by requiring the conic determined by  $\cos \theta = \kappa$ , along which the source angular distribution maintains its peak value, to coincide with the corresponding hyperbolic arc. Finally,  $\Sigma$  is replaced by an approximation,  $\Sigma_0$ , more convenient for calculation: the boundaries of  $\Sigma_0$  are segments of ellipses (sides) and hyperbolas (top and bottom) "parallel" to the curvilinear axes of the natural coordinate system for the problem.

We remark, finally, that the use of natural coordinates should simplify mesh size problems encountered in programming the surface integrals for numerical integration, at a considerable savings in computer time and cost. Small mesh sizes can be chosen for the  $v$ -integration, probing the rapid variation of the source function through the peak near  $\cos \theta = \kappa$ , and large mesh sizes will suffice for the  $v$ -integration since the  $v$ -dependence of the integrand is relatively weak.

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